

Open Problem: Let  $E \subset \mathbb{N}$  be syndetic. is  $E$  GP-rich?

Not known:  $x, xq, xq^2 \in E$  !!!

even  $x, xq^2 \in E$  not known.

↓

$E/E$  contains a square

#3:  $N = \bigcup_{i=1}^r (S - t_i) \Rightarrow S - t_i$  is AP-rich  $\Rightarrow S$  is AP-rich

so by finitistic vdw,  $S \cap T$  is AP-rich.

Ex: does vdw follow from the fact that any syndetic set is AP-rich

Call a set  $A \subset \mathbb{N}$  piecewise syndetic if it contains  $S \cap T$  where  $T$  is thick &  $S$  is syndetic.

Ex: show if  $N = \bigcup_{i=1}^r C_i$  then one of  $C_i$  is pws.

Ex: If  $E$  is pws &  $E = \bigcup_{i=1}^r C_i$  then one  $C_i$  is pws.

Is it true that  $\lambda(A) > 0 \Rightarrow A \supset$  a shift of an IP set?

#6:  $d(A \cap A^{-n_1}) = \frac{1}{4}$  (in particular,  $A \cap A^{-n_1} \neq \emptyset$ ).

$$d(\underbrace{(A \cap A^{-n_1}) \cap ((A \cap A^{-n_1})^{-n_2})}_{A \cap A^{-n_1} \cap A^{-n_2} \cap A^{-n_2-n_1}}) = \frac{1}{16}$$

$$A \cap A^{-n_1} \cap A^{-n_2} \cap A^{-n_2-n_1}$$

Reading: Thru ch 11 by Friday

Ex: does  $P^{-1}$  contain an IP set?