Lec 1/30

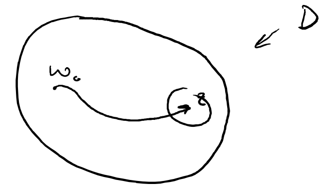
Tuesday, January 30, 2018 10:41

Lemma 4: Suppose f is cts in a plane domain $D \subset \mathbb{C}$ and that $\forall$ closed path $\gamma \subset D$ we have

$$\int_\gamma f(z)\, dz = 0.$$

Then f has a primitive F

Proof: take fixed $w_0 \in D$, and $\forall z$ a pws path α from w_0 to z contained entirely in D.



Define $F(z) = \int_\alpha f(z')\, dz'$

This is uniquely defined bc if β also from w_0 to z

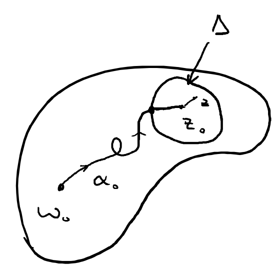
then $$\int_\alpha f(z')\, dz' - \int_\beta f(z')\, dz' = \int_{\alpha-\beta} f(z')\, dz' = 0.$$

Now need to show $F'(z_0) = f(z_0)$. Since D is a domain, $\exists\ \Delta(z_0, \delta) \subseteq D$. We take

α_0 connects w_0 to z_0,

$$F(z) = \int_{\alpha_0} f(z')\, dz' + \int_{z_0}^{z} f(z')\, dz'$$

could replace by



we know $G(z) = \int_{z_0}^{z} f(z')\, dz'$ is a primitive for f in Δ.

i.e. $G'(z) = f(z)$ (by Last Lemma)

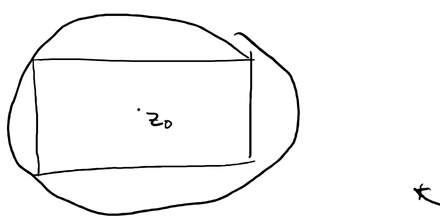
So $F'(z) = 0 + G'(z) = f(z)$.

Local Cauchy Theorem:

Suppose Δ is an open disk in $\mathbb{C}$ where f is continuous & Analytic in $\Delta \setminus \{z_0\}$. Then $\forall$ pwsc γ, $\int_\gamma f(z)\,dz = 0$.

Proof from lemma 2, $\int_{\partial R} f(z)\,dz = 0$. So Lemma 3 says $\exists F$ a primitive for z s.t. $F'(z) = f(z)$. Thus $\int_\gamma f(z)\,dz = 0$.

$$\int_{\partial R} \frac{1}{z - z_0}\,dz$$ where R is a rectangle w/ z_0 at the center.



Choose a circle as shown. Can subtract off bits to show the integral is

$$\int_{|z-z_0|=1} \frac{1}{z-z_0}\,dz = i2\pi$$