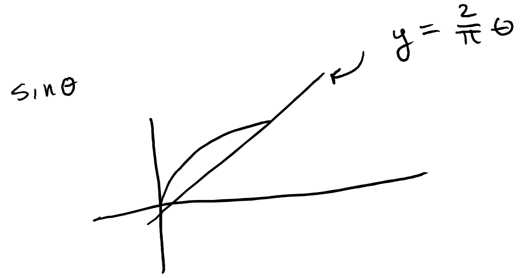
Lec 1/26

Friday, January 26, 2018 10:52

finding an upper bound for $|I|$ w/

$$I = \int_\gamma e^{iz^3}\,dz \quad \text{with} \quad \gamma(t) = re^{it} \quad 0 \le t \le \pi/6$$

$$|I| \le \frac{r}{3}\int_0^{\pi/2} e^{-r^3 \sin\theta}\,d\theta \qquad \text{by various args}$$



$$\le \frac{r}{3}\int_0^{\pi/2} e^{-\frac{2}{\pi}r^3\theta}\,d\theta$$

$$-\frac{r/3}{(2/\pi)r^3}\, e^{-\frac{2}{\pi}r^3\theta}\Big|_0^{\pi/2}$$

Thm

If $f(z) = F'(z)$ for some analytic F in some domain $U \subset \mathbb{C}$

Then $\int_\gamma f(z)\,dz = F(\gamma(b)) - F(\gamma(a))$

for any p.w.s. path γ.

Pf

$$\int_\gamma f(z)\,\partial z = \int_a^b f(\gamma(t))\,\gamma'(t)\,\partial t$$

$$= \int_a^b F'(\gamma(t))\,\gamma'(t)\,\partial t$$

$$= \int_{\gamma(a)}^{\gamma(b)} F'(u)\,\partial u$$

$$= F(\gamma(b)) - F(\gamma(a))$$

Calculate

$$\int_\gamma \frac{1}{z(z+2)}\,\partial z \qquad \text{where} \quad \gamma(t) = e^{it}$$

$$= \int_\gamma \frac{1}{2}\left(\frac{1}{z} - \frac{1}{z+2}\right)\partial z$$

$$= \frac{1}{2}\int_\gamma \frac{dz}{z} - \frac{1}{2}\int_\gamma \frac{dz}{z+2}$$

$$= \frac{1}{2} \int_{\gamma} \frac{dz}{z} - \frac{1}{2} \int_{\gamma} \frac{dz}{z+2}$$

$$= \frac{1}{2} \left(\mathrm{Log}\, z \right) - \frac{1}{2} \left(\mathrm{Log}\, (z+2) \right) \Big|_{1}^{1}$$

regular Log
discontinuous thru path, <u>not</u> 0.

$$= \pi i$$

use Log w/ branch cut