Lec 1/23

Tuesday, January 23, 2018 10:30

$f(z) = \sin z$

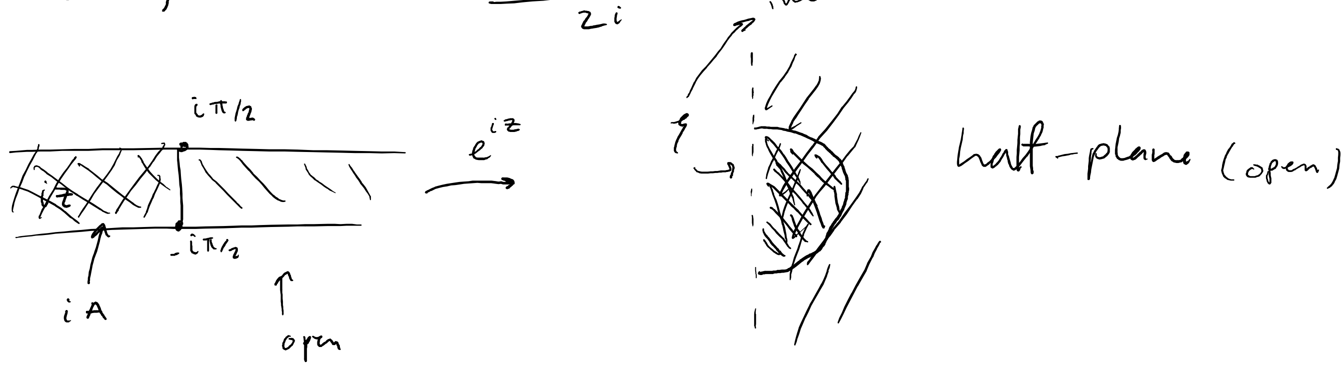
Lower half-strip maps to lower half strip

if we remove middle lines, sin is univalent

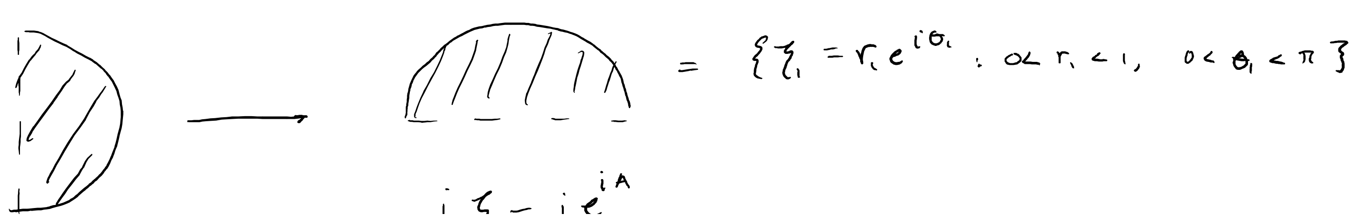
What is inverse?

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i}$$

let $\zeta = e^{iz}$ $\quad w = \dfrac{\zeta - 1/\zeta}{2i}$



$$w = \frac{\zeta - 1/\zeta}{2i} = -\left(i\zeta + \frac{1}{i\zeta}\right)\frac{1}{2}$$



ξ

$$i\xi = ie^{iA}$$
$$\begin{array}{c} \| \\ \xi_1 \end{array}$$

$$w = -\frac{1}{2}\left(\xi_1 + \frac{1}{\xi_1}\right)$$

Joukowski's map.

put $\xi_1 = r_1 e^{i\theta_1}$

$$w = -\frac{1}{2}\left(r_1 e^{i\theta_1} + \frac{1}{r_1} e^{-i\theta_1}\right)$$

$$= \left(r_1 \cos\theta_1 + \frac{1}{r_1}\cos\theta\right) + i\left(r_1 \sin\theta_1 - \frac{1}{r_1}\sin\theta_1\right)$$

if $r_1 = c_1$ we get an ellipse.

$$\frac{u^2}{\text{somethin}^2} + \frac{v^2}{\text{somethin}^2} = 1$$

if $\theta_1 = c_1$, we get a hyperbola.

$$2w\xi_1 = \xi_1^2 + 1$$

$$\xi_1 = -w \pm \sqrt{w^2 - 1} = -w \pm i\sqrt{1 - w^2}$$

choose principal branch

The choice of $\xi_1 = -w + i\sqrt{1-w^2}$ is the correct inverse

can prove $\mathrm{Im}\{-w + i\sqrt{1-w^2}\} > 0$.

$$\sin z = w$$

$$\frac{e^{iz} - e^{-iz}}{2i} = w$$

$$\xi = e^{iz}$$

$$\frac{\xi^2 - 1}{2i\xi} = w$$

$$\xi = \frac{2iw \pm \sqrt{4 - 4w^2}}{2}$$

$$= iw \pm \sqrt{1 - w^2}$$

$$z = -i \operatorname{Log}(\xi)$$

$z = -i \operatorname{Log}\left(iw + \sqrt{1-w^2}\right)$ is an inverse function.

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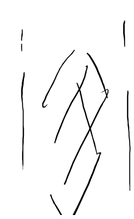
determine mapping properties of $f(z) = \tan z$ for domain

$D = \{ z : -\frac{\pi}{2} < \mathrm{Re}\, z < \frac{\pi}{2} \}$ =



& determine appropriate formula for f^{-1} s.t. $f^{-1}(f(D)) = D$.

determine $\frac{d}{dz} \arctan z$

$$\tan z = \frac{\dfrac{e^{iz} - e^{-iz}}{2i}}{\dfrac{e^{iz} + e^{iz}}{2}} = \frac{i(1 - e^{i2z})}{1 + e^{i2z}}$$

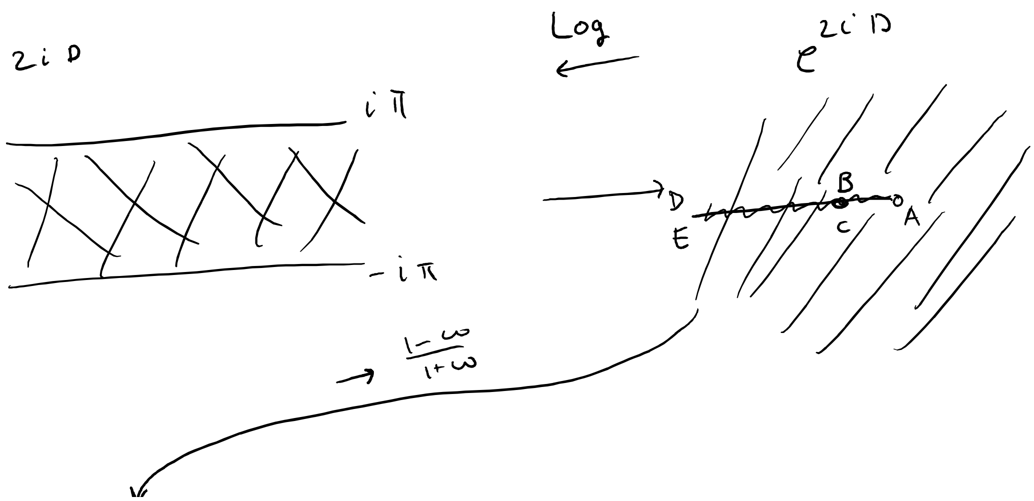
$f_1(z) = 2iz$

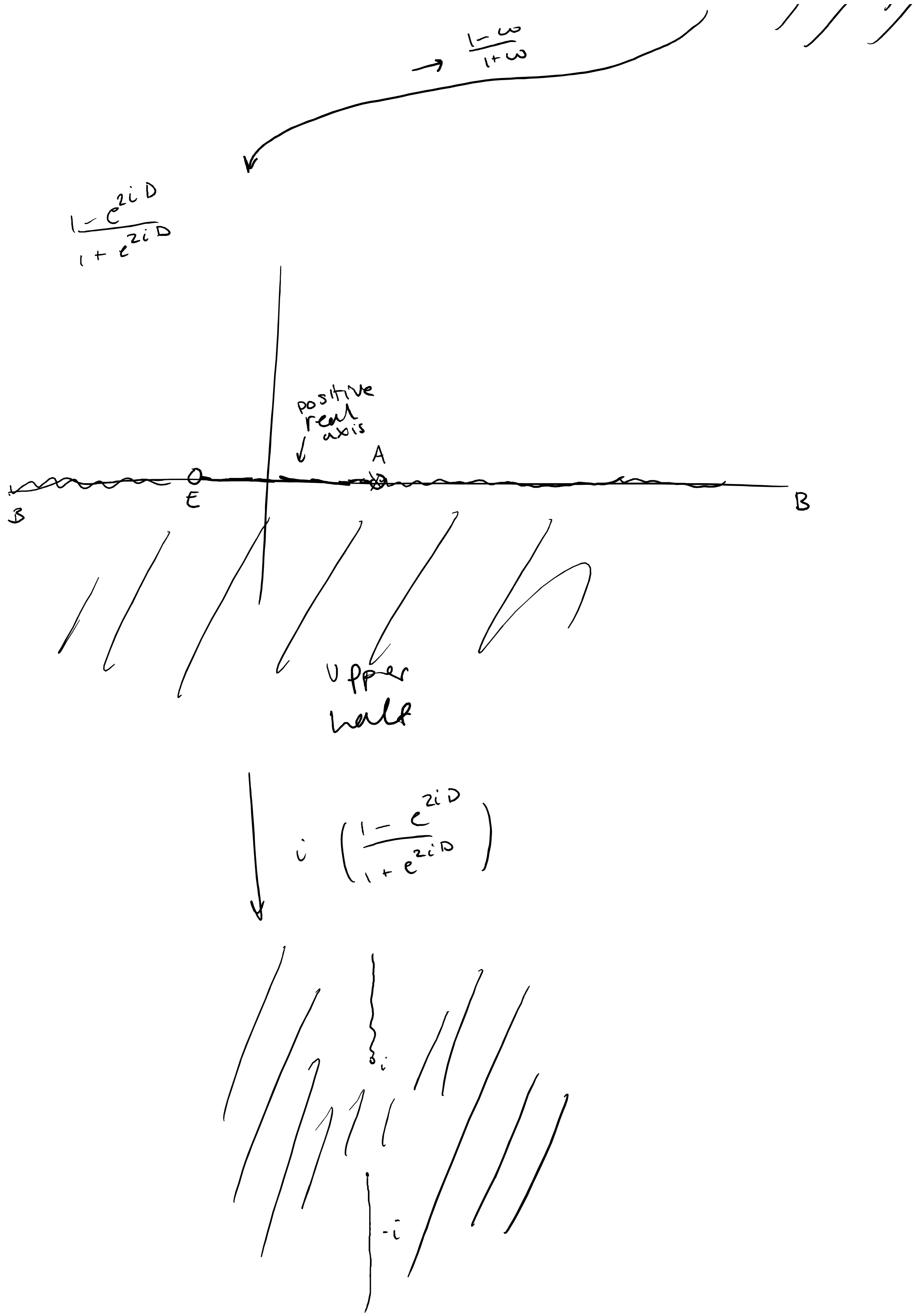
$f_2(z) = e^z$

$f_3(z) = \frac{1-z}{1+z}$

$f_4(z) = iz$

so $\tan = f_4 \circ f_3 \circ f_2 \circ f_1$





$$\arctan = f_1^{-1} \circ f_2^{-1} \circ f_3^{-1} \circ f_4^{-1}$$

$$\arctan w = -\frac{i}{2} \operatorname{Log}\left(\frac{1+iw}{1-iw}\right)$$

$$\frac{d}{dw} \arctan w = \frac{1}{\sec^2 z} = \frac{1}{\sec^2(\arctan z)}$$

$$= \frac{1}{1+\tan^2 z} = \frac{1}{1+w^2}$$