Lec 1/16

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Thm Let $z_0, z \in D$, a domain. Then $\exists$ a polygonal line from z_0 to z.

Pf Define $U = D(z_0)$, $V = \bigcup_{z \neq z_0 \text{ distinct}} D(z)$. $D \subset U \cup V$ and $U \cap V = \emptyset$

thus either $D = \emptyset$ or $V = \emptyset$.

Thm Suppose $U \subset \mathbb{C}$ is open and A is a connected subset of U.

then A is contained in the same connected component of U.

Thm if $A \subset \mathbb{C}$ is connected & f is cts, $f(A)$ is connected.

Pf use preimage to show A disconnected if $f(A)$ disconnected.

Assume $f(A) \subset U \cup V$ (open), $U \cap V = \emptyset$, $U \cap f(A) \neq \emptyset \neq V \cap f(A)$.

Let $C_U = f^{-1}(U)$, $C_V = f^{-1}(V)$. then $C_U \cup C_V \supset A$, $C_{U/V} \cap A \neq \emptyset$,

and $C_U \cap C_V \cap A \neq \emptyset$. thus A is disconnected.