Convex sets, convex hull.

A set $S = \{x_0, x_1, \dots, x_n\} \subseteq \mathbb{R}^k$ is in <u>general position</u> if it is not contained in any affine subspace of dimension $< n$.

S in gen. pos. $\Rightarrow n \leq k$.

Def for a finite set $\underset{\{x_0, \dots, x_n\}}{S}$ in general position, the simplex associated to S is $\sigma(S) = \mathrm{Conv}(S)$.

The points x_i are called vertices of $\sigma(S)$.

$\forall$ non-empty subset $T \subseteq S$ the simplex $\sigma(T) \subseteq \sigma(S)$ is called a face of $\sigma(S)$.

If $|T| = 2$, we call $\sigma(T)$ an edge of $\sigma(S)$.

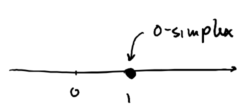
The dimension of $\sigma(S)$ is $\dim(\sigma(S)) = n = |S| - 1$.

In general, a subset $\sigma \subseteq \mathbb{R}^k$ is called a simplex if $\sigma = \sigma(S)$ for some S in gen. pos.

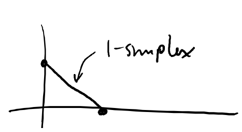
The standard n-simplex is $\sigma(S)$ where

$$S = \{(1,0,\dots,0), \dots, (0,\dots,0,1)\} \subseteq \mathbb{R}^{n+1}$$

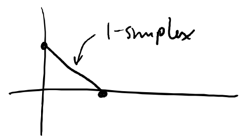
eg. $n = 0$



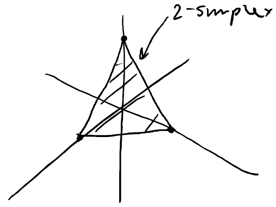
$n = 1$



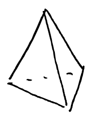
$n=1$



$n=2$



$n=3$


$\subseteq \mathbb{R}^4$

Simplicial Complexes

A (geometric) simplicial complex is a finite collection of simplices $X = \{\sigma_i\}_{i=1}^{N}$

w/ each $\sigma_i \subseteq \mathbb{R}^n$ satisfying

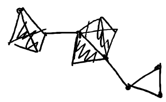
(a) $\forall\, \sigma \in X$, all faces of σ are in X.

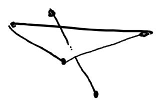
(b) $\forall\, \sigma, \tau \in X$, if $\sigma \cap \tau \neq \emptyset$, $\sigma \cap \tau$ is also a simplex which is a face of both σ and τ.

Remarks:

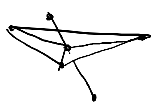
- we frequently conflate the collection & its union.
- Intuitively, X is a set obtained from gluing together simplices along faces.

Ex: Geometric realization of a graph.

Ex: 

BADEX: 

Triangulate $\Big\downarrow$

GOODEX: 

An abstract simplicial complex is

$\rightsquigarrow$

$$V = \{a, b, c, d, e\}$$

$$\Sigma = \left\{ \begin{array}{l} \{a\}, \{b\}, \{c\}, \{d\}, \{e\}, \\ \{a,b\}, \{b,c\}, \{a,c\}, \{c,d\}, \{c,e\}, \\ \{a,b,c\} \end{array} \right\}$$