Lec 3/8

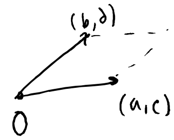
Wednesday, March 8, 2017 09:11

linear map $G : \mathbb{R}^2 \to \mathbb{R}^2$

linear: $G(u+v) = G(u) + G(v)$, $G(cu) = c\,G(u)$

$G \approx \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $G\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$

Image of unit square is parallelogram w/



area $= \overset{\text{ish}}{\widetilde{(a,c) \times (b,d)}}$

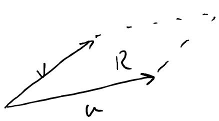
$= \left|\det \begin{pmatrix} a & b \\ c & d \end{pmatrix}\right|$

if $e_2 \uparrow \circlearrowleft$ ccw e_1 $\xrightarrow{G}$ $e_1 \circlearrowright$ ccw e_2 then det is negative.

"right handed" or "left handed"

why determinant = volume. Volume (box of v_1, v_2, v_3) $= \left|\det\left(v_1 | v_2 | v_3\right)\right|$

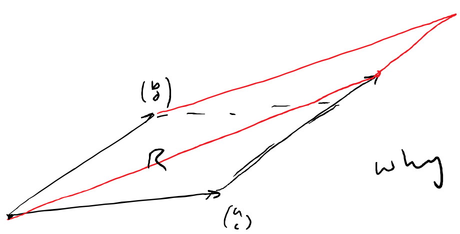
can extend $\times$ to n dimensions.



area $R = \sqrt{|u|^2|v|^2 - (u\cdot v)^2}$

note: $(u\cdot v)^2 \leq |u|^2|v|^2$

area



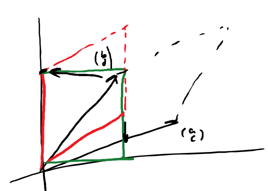
area R = area R

so $c_1 \leftrightarrow s c_2$ does not change det.

why area $R = ad - bc$

$\begin{pmatrix} a + xb & b \\ c + xd & d \end{pmatrix}$ (same det as $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$)

$c_1 \leftrightarrow c_1 + x c_2$ so add



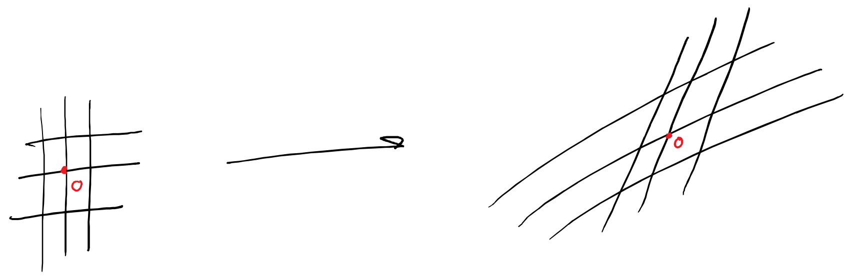
$1 \to 2 \to 3$

row-reduction

$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \longrightarrow \begin{pmatrix} r & 0 \\ 0 & s \end{pmatrix}$

row - reduction

$\mathbb{R}^2 \longrightarrow \mathbb{R}^2$

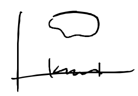


$\text{area}(G(R)) = |\det(G)|\, \text{area}(R)$ if R is made of squares (easy)

and if not (hard, take limit)

↳ as long as R is contented / Jordan measurable.

$G\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ 0 \end{pmatrix} \approx \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$



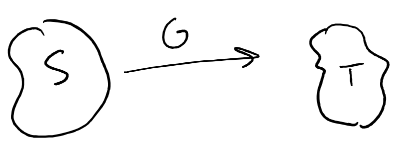
$\det(G) = 0.$

assume G invertible $\Leftrightarrow \det(G) \neq 0.$ $\Delta = |\det(G)|$

Prop S measurable $\Rightarrow$ $\text{area}(G(S)) = \Delta\, \text{area}(S)$ ($G(S)$ measurable)

$$\iint_{G(S)} 1 \, dx\, dy = \Delta \iint_S 1 \, du\, dv$$

f of class C' on region S.



$T = G(S)$

$$\iint_S f(x,y)\, dx\, dy = \Delta \iint f(u,v)\, du\, dv$$ where $\begin{pmatrix} u \\ v \end{pmatrix} = G\begin{pmatrix} x \\ y \end{pmatrix}$

example

matrix (G) = Jacobian

matrix (G) jac

$G\binom{x}{y} = \binom{u}{v}$ $\begin{cases} u = x + 3y \\ v = 2x - y \end{cases}$ $\qquad f(x,y) = x^2 y + \sin(xy)$

$$\iint_S f(x,y)\,dx\,dy = \iint_{G^{-1}(S)} f(u,v)\,\Delta\,du\,dv$$

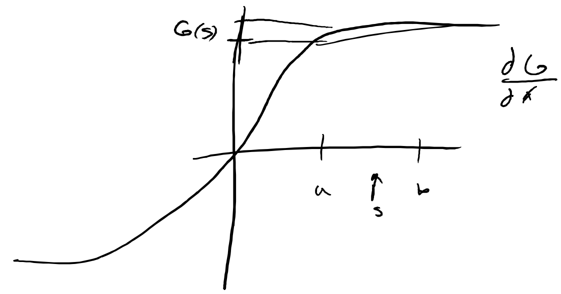
$$\Delta \to \left|\frac{\partial(x,y)}{\partial(u,v)}\right|$$

"put area change factor in each little piece"

$x = r\cos\theta \qquad y = r\sin\theta \qquad$ (polar)

S in x,y plane

T in r,θ plane

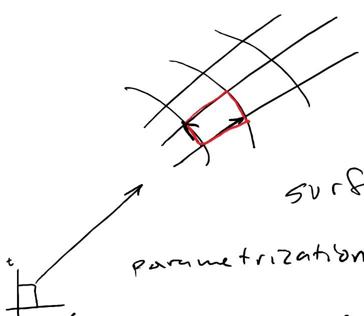


non-linear.

$$\iint_S f(x,y)\,dx\,dy = \iint_T f(r,\theta)\,(??)\,dr\,d\theta$$

$$\frac{\partial(x,y)}{\partial(r,\theta)} = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} = r.$$

Parallel to u-sub.



surface S.

parametrization.

$$\text{area} = \frac{\partial f}{\partial s} \times \frac{\partial f}{\partial t}$$

3 ways: $f(x,y) = z$

$\vec{f}(u,v) = (x,y,z)$

$F(x,y,z) = 0$

↑ not so good for integration.

$$\iint_{\text{sphere}} f = \iint_{(s,t)} f \left|\frac{\partial f}{\partial s} \times \frac{\partial f}{\partial t}\right| ds\,dt.$$

↳ use determinant if not 2 vectors in $\mathbb{R}^3$.

this is the same as above.