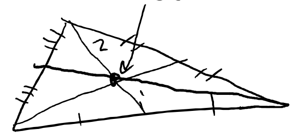
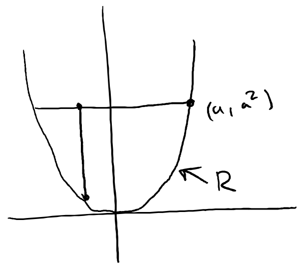
Lec 3/7

Tuesday, March 7, 2017 09:18

Centroid / center of mass

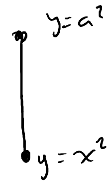




y-coord of centroid: $\dfrac{\iint y\,dx\,dy}{\iint 1\,dx\,dy}$

$$\iint_R 1\,dx\,dy = \text{area}(R)$$

Given $x \in [-a, a]$, get slice



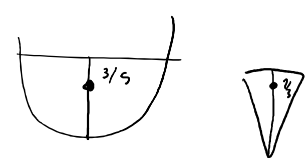
$$= \int_{-a}^{a} \underbrace{\int_{x^2}^{a^2} 1\,dy}_{a^2 - x^2}\,dx$$

$$= \int_{-a}^{a} (a^2 - x^2)\,dx = \left(a^2 x - \tfrac{1}{3}x^3\right)\Big|_{-a}^{a} = 2\left(a^3 - \tfrac{1}{3}a^3\right) = \tfrac{4}{3}a^3$$

$$\int_{-a}^{a}\int_{x^2}^{a^2} y\,dx = \int_{-a}^{a}\left(\frac{a^4}{2} - \frac{x^4}{2}\right)dx = \frac{2}{2}\left(a^5 - \tfrac{1}{5}a^5\right) = \tfrac{4}{5}a^5$$

got this w/ fubini

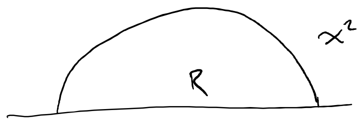
So y-coord is $\frac{3}{5}a^2$.



x-coord is 0 since symetric.

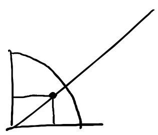
Verify $\iint x\,dx\,dy = 0$

Verify $\iint_R x\,dx\,dy = 0.$



$x^2+y^2=1,\ y \geq 0.$

$\iint_R 1\,dx\,dy = \frac{\pi}{2}$ $\quad \iint_R y\,dy\,dx = \int_{-1}^{1}\int_0^{\sqrt{1-x^2}} y\,dy\,dx = \int_{-1}^{1} \frac{1-x^2}{2}\,dx = \frac{2}{2}\left(1-\frac{1}{3}\right) = \frac{2}{3}$



So y-coord of centroid is : $\frac{2/3}{\pi/2} = \frac{4}{3\pi}$

$dx\,dy = r\,dr\,d\theta$
↳ Jacobian.

$\iint_R y\,dy\,dx = \int_0^{\pi}\int_0^1 (r\sin\theta)\, r\,dr\,d\theta$

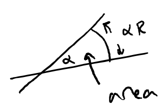
$= \int_0^{\pi} \sin\theta\,d\theta \int_0^1 r^2\,dr$

$= 2 \cdot \frac{1}{3}$

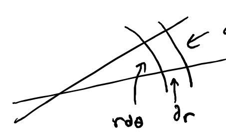
$= \frac{2}{3}$

πR^2 = area of sector

$\frac{\alpha}{2\pi}$ = ratio



area = $\frac{1}{2}\alpha R^2$



← area of little region

$\underbrace{dx\,dy = r\,dr\,d\theta}$

"area change factor" $= r$

$$\int_a^b f(x)\,dx = \int_{x^{-1}(a)}^{x^{-1}(b)} f(x(t))\,x'(t)\,dt$$

replace $x(t)$ w/ $g(t)$.

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$x = x(t)$

$dx = x'(t)\,dt$

chain rule.

$$\int f(x)\,dx = \int f(x(t))\,\frac{dx}{dt}\,dt$$

For change of variables, $\displaystyle\int_a^b f(x)dx = \int_c^d f(g(t))\,g'(t)\,dt$

$x = g(t)$ on $[a,b]$

want g to be injective (one-to-one)

diffable

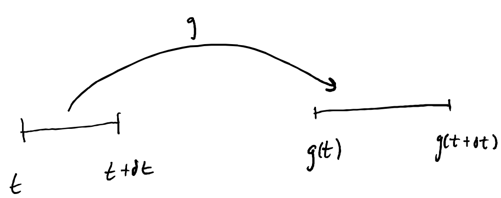
$g:[a,b] \longrightarrow \mathbb{R}$

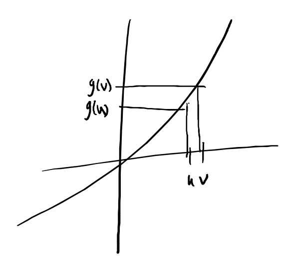
g must be monotone (strictly)

$x = a \Leftrightarrow t = c$, etc.

Increasing / decreasing could pose problems, but g' would be negative.

so take $\displaystyle\int_{[a,b]} f(x)dx = \int_{g'([a,b])} f(g(t))\,|g'(t)|\,dt$





$$\frac{g(v) - g(u)}{u - v} \longrightarrow g'(u)$$

how much is square stretched in $\mathbb{R}^2$? Jacobian or determinant of that.