Lec 3/6

Monday, March 6, 2017 09:09

$A \subseteq \mathbb{R}^n$ set

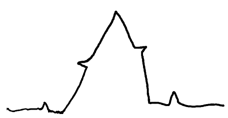
Jordan Content

content = 0

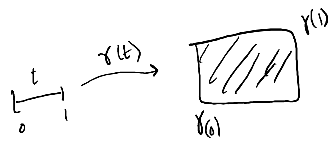
measure = 0

Lebesgue

Lengths of Curves.



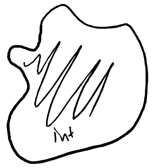
Cauchy sequence of functions $f_n : [0,1] \to \mathbb{R}^2$



continuous γ maps $[0,1]$ to the square.

Jordan: Curve in plane

$\gamma : [0,1] \to \mathbb{R}^2$ jordan curve never crosses itself except $\gamma(0) = \gamma(1)$.



Cts closed curve.

then $\iint_{Int} f(\vec{x})\, dV$ exists.

Jordan Curve Theorem.

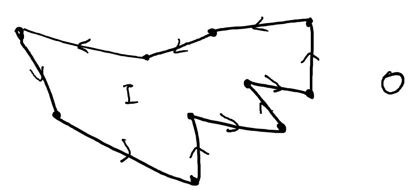
a curve like this has an interior & an exterior.

easy case: polygons rather than continuous curves.

Jordan polygon:

I & O open & connected.

$I \cup O$ disconnected.



easier since a line crossing a polygon crosses curve finitely many times.

$$\iint_D f(x,y)\,dx\,dy = \iint_R (f\cdot\chi_D)(x,y)\,dx\,dy$$

$V = C^1[0,1]$ infinite dimensional

1
x
x^2
x^3
x^4
$\vdots$

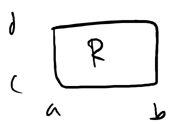
$g: C^1[0,1] \to \mathbb{R}$

$g(f) = f(0)$

$g(f) = \int_0^1 f(t)\,dt$

etc.

Double integrals can be iterated integrals w/ some conditions on f.



$$\underbrace{\int_a^b\int_c^d f(x,y)\,dy\,dx}_{\text{if this exists, this holds.}} = \iint_R f = \underbrace{\int_c^d\int_a^b f(x,y)\,dx\,dy}_{\text{" "}}$$

i.e. $f_x(y) = f(x,y)$ is integrable $\forall x \in [a,b]$, $H(x) = \int_c^d f(x,y)\,dy$ is integrable on $[a,b]$.

Note: restrict to bounded functions.

suppose $f(x,y)$ cts on every line through $(0,0)$, $f(0,0)=0$. is f cts at $(0,0)$?

No, $f(x) = \begin{cases} 1 & \text{if } y = x^2,\ x \neq 0. \\ 0 & \text{else} \end{cases}$