Lec 3/3

Friday, March 3, 2017 09:08

Read for monday: 4.3, 4.4.

• euler substitutions

$$\int_a^b f(x)\,dx \qquad x = g(t)\,; \qquad dx = g'(t)\,dt$$

$$\int_a^b\int_c^d f(x,y)\,dx\,dy = \int_a^b \left(\int_c^d f(x,y)\,dx \right) dy = \int_c^d \left(\int_a^b f(x,y)\,dy \right) dx$$

(fubini)

$$\frac{d}{dx} \int_{g_1(x)}^{g_2(x)} f(t)\,dt = f(g_2(t))g_2'(t) - f(g_1(t))g_1'(t)$$

$$\int_{-\infty}^{\infty} f(x)\,dx \overset{\text{hopefully}}{=} \lim_{(A,B)\to(-\infty,\infty)} \int_A^B f(x)\,dx$$

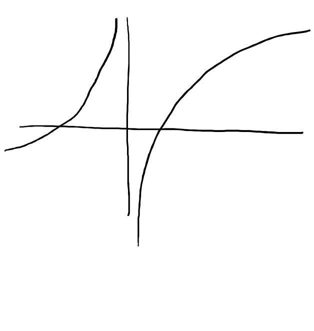
$\int_{-\infty}^{\infty} \frac{1}{x}\,dx$ should be zero. We have to take it to be $\underbrace{\lim_{A\to\infty} \int_{-A}^{A} \frac{1}{x}\,dx}_{\text{Symmetrizing.}}$

$$\int \frac{P_1(x)}{P_2(x)}\,dx$$

↳ integrable in elementary functions by Partial Fraction decomposition

Boole's transformation/substitution.

$$X \to X - \frac{1}{x} \quad , \quad x \in \mathbb{R}\setminus\{0\}$$



$$\boxed{\int_{-\infty}^{\infty} f\left(x-\tfrac{1}{x}\right)dx = \int_{-\infty}^{\infty} f(x)\,dx}$$ ← Bonus exercise

assuming f is nice enough ($\int_{-\infty}^{\infty} f(x)\,dx$ exists w/ $A,B \to \infty$ indep).

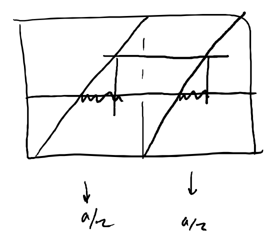
$f^{-1}((0,a]) = \{x : f(x) \in (0,a]\}$

→ where $f(x) = x - \frac{1}{x}$

exercise: $m\left(f((0,a])\right) = a$. (same as $f(x) = x$)

help

and



approximate f by limit of convenient functions
(in integral)

Weierstrass: $\forall f \in C[a,b], \forall \varepsilon > 0, \exists P \in \mathbb{R}[x]$ s.t. $\max_{x \in [a,b]} |f(x) - P(x)| < \varepsilon$

Note: not true on the real line.

Why taylor formula doesn't work: interval of convergence may be smaller than $[a,b]$.

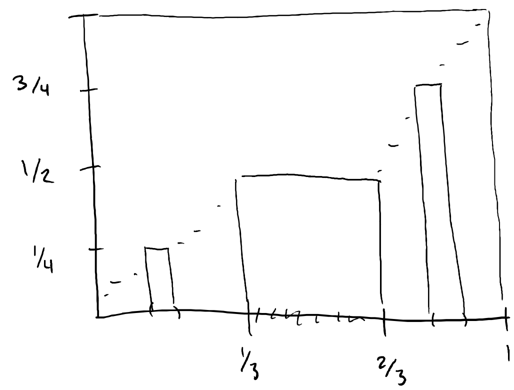
$f_n(x) \longrightarrow f(x)$ pointwise or uniformly in sup norm

Uniform: $\forall \varepsilon > 0\ \exists N$ s.t. $\forall n > N$, $\sup_x |f(x) - f_n(x)| < \varepsilon$.

If $f_n : [0,1] \longrightarrow \mathbb{R}$ cts and $f_n \to f$ uniformly then f continuous. (exercise)

$\lim \int_a^b f_n(x)\, x \overset{?}{=} \int_a^b \lim f_n(x)\,dx$ What if f is badly discontinuous?

$$\int_a^b f'(x)\,dx \overset{?}{=} f(b) - f(a)$$



On each adjacent interval it has constant value. diff-able almost everywhere.

Cantor Stairs.

defined on $[0,1] \setminus C$ (Cantor set) define on C by "continuity"

exercise: complement of any set of measure zero is dense

exercise: What is the length of the graph? probably 2.