Lec 3/20

Monday, March 20, 2017 09:17

Midterm 2: Next wed: 3, 4, 5.1, 5.2 ?

4.6: limit comparison for improper integrals of 1 var:

Analogous to L.C. for series.

$\sum_{n=c}^{\infty} a_n$, $\sum_{n=c}^{\infty} b_n$ both series of positive terms, $\lim_{n\to\infty} \frac{a_n}{b_n} = L >$ then the series both cvg or div.

f $L=0$ then b_n converges $\Rightarrow$ a_n converges

$\int_c^{\infty} f$, $\int_c^{\infty} g$, f, g both cts and nonnegative, $\lim_{x\to\infty} \frac{f(x)}{g(x)} - L >$ then the integrals both cvg or div.

LCT for integrals, V1:

$f, g \geq 0$ and cts on $[a, \infty)$, $\lim_{x\to\infty} \frac{f(x)}{g(x)} = L > 0$ then $\int_a^{\infty} f$ cvgs $\Leftrightarrow$ $\int_a^{\infty} g$ cvgs.

If $L=0$ then $\int_a^{\infty} g$ cvgs $\Rightarrow$ $\int_a^{\infty} f$ cvgs.

Proof sketch: use comparison test. $\left((L \pm \varepsilon)g \text{ with } f\right)$

LCT for integrals, V2:

f, g cts on $[a, b)$, $b \neq \infty$. $\lim_{x\to b^-} f(x) = \infty = \lim_{x\to b^-} g(x)$, $\lim_{x\to b^-} \frac{f(x)}{g(x)} = L > 0$

then $\int_a^b f$ cvgs $\Leftrightarrow$ $\int_a^b g$ cvgs. if $L=0$ then $\int_a^b g$ cvgs $\Rightarrow$ $\int_a^b f$ cvgs.

Proof sketch same as above.

4.6 #2 b

$$\int_0^1 \frac{dx}{x^{1/2}(x^2+x)^{1/3}} \quad \left(\text{compare to } \left(\frac{1}{x^{5/6}}\right)\right)$$

4.7 #4

for a use polar coordinates.

5.1: Integrals over Curves

Arc length of a parametric curve.

$$\vec{g} : [a,b] \longrightarrow \mathbb{R}^n$$

Assume $\vec{g}$ is C^1 (extends to a C^1 function on $(a-\varepsilon, b+\varepsilon)$).

Physics approach: $\vec{g}'(t)$ is velocity at time $t \in [a,b]$.
$|\vec{g}'(t)|$ is speed.

length of curve is limit over partitions of polygonal approximations:

$$\inf_{\substack{P \text{ partitions} \\ \text{of } [a,b]}} U(|\vec{g}'(t)|, P) = \int_a^b |\vec{g}'(t)|\, dt$$

geometric approach: tomorrow.