Lec 3/1

Wednesday, March 1, 2017 09:09

Theorem any monotone & bounded function $f : [a, b] \to \mathbb{R}$ is Riemann Integrable.

Remark In any jump interval there is a rational number. These are disjoint.
there are only countably many points where it's uncountable

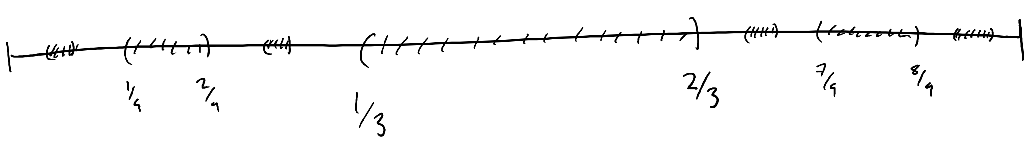
Is it True that any countable subset of $S \subseteq \mathbb{R}$ can be the set of points of discontinuity of some monotone function

$\boxed{\text{YES}}$ (bonus exercise)

Theorem for any monotone function f the set of points of nondiffability has measure zero.

f is diffable "almost everywhere"

Uncountable measure zero set? Yes, Cantor set.



... etc. $K_1, K_2, \ldots$

Cantor set $C = \bigcap_{n=1}^{\infty} K_n$

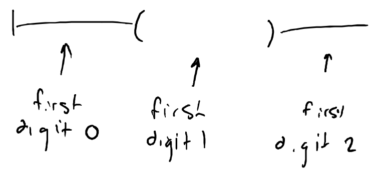
$n=1$

claim: ① $m(C)=0$ all endpoints stay.

② C is uncountable

any $t \in [0,1]$ has a ternary representation.

$t = \sum \frac{t_i}{3^i}$, $t_i \in \{0,1,2\}$.



second iteration removes points w/ second digit 1.

So $C = \{ t = \sum \frac{t_i}{3^i} : t_i \in \{0,2\} \} = \{0,2\}^{\mathbb{N}}$ uncountable.

to show $m(C)=0$, any $K_n \supseteq C$ and K_n is made of finitely many intervals of total length $(\frac{2}{3})^n \to 0$. (bonus: prove this)

<u>Theorem</u> (Lebesgue): a bounded $f : [a,b] \to \mathbb{R}$ is Riemann integrable iff the set of points of discontinuity of f has measure zero.

Measure zero in $\mathbb{R}^2$ or n.

$S \subseteq \mathbb{R}^2$ has $m(S)=0$ if $\exists \{R_i\}$ s.t. $S \subseteq \bigcup_{i=1}^{n} R_i$ and $\sum \text{Area}(R_i) < \varepsilon$.

m(any "nice" curve) $= 0$ (bonus exercise)