Lec 2/22

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Lemma 1: If $\alpha \in C[a,b]$ and $\int_a^b \alpha(x)h(x)\,dx = 0 \;\; \forall\, h \in C[a,b]$ w/ $h(a)=h(b)$, then $\alpha(x)=0$.

Proof: Consider $h(x) = \alpha(x)\,(x-a)\,(b-x)$. $h(a)=h(b)=0$, and $(x-a)(b-x)>0$ on (a,b), and $(\alpha(x))^2 \geq 0$ on $[a,b]$ so for $\int_a^b (\alpha(x))^2 (x-a)(b-x)\,dx = 0$ we must have $\alpha(x)=0$.

≥ 0 on $[a,b]$

Implicit function theorem

notion of a curve.

Locus of an equation $F(x,y)=0$.

Claim: $x - y - y^5 = 0$ can be $x(y)$ or $y(x)$ (a function globally).

Could be: $F_i(x_1, \dots, x_n, y_1, \dots, y_m)$ for $i = 1, \dots, k$.

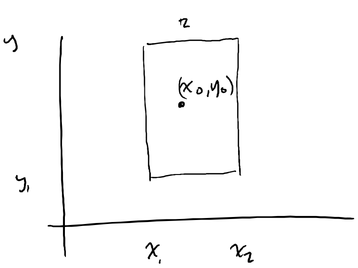
(circle divides plane in 3)

look up: Jordan Curve Theorem.

What we assume of $F(x,y)$: (Perhaps not optimal but sufficient).

- $F \in C^1$
- point (x_0, y_0) satisfies $F(x_0, y_0) = 0$.
- $F_y(x_0, y_0) \neq 0$

Then $\exists$ rectangle



where $F(x,y)=0$ defines a continuous function $y = f(x)$ s.t. $y_0 = f(x_0)$ & $F(x, f(x)) = 0$ $\forall\, x \in [x_1, x_2]$. also f is diff'able.

moreover, $y' = f'(x) = -\dfrac{F_x}{F_y}$

$$F(x+h, y+k) = F(x,y) + hF_x(x,y) + kF_y(x,y) + \varepsilon_1 h + \varepsilon_2 k$$ and $\varepsilon_1, \varepsilon_2 \to 0$ as $h,k \to 0$

and in our situation $k \to 0$ as $h \to 0$.

(h,k) in rectangle around origin.

and $F(x,y) = 0$

$F(x+h, y+k) = 0$.

$$\Rightarrow 0 = hF_x + kF_y + \varepsilon_1 h + \varepsilon_2 k$$

$$0 = \frac{F_x}{F_y} + \frac{k}{h} + \frac{\varepsilon_1}{F_y} + \frac{\varepsilon_2 k}{hF_y};$$

$$\left(1 + \frac{\varepsilon_2}{F_y}\right)\frac{k}{h} + \frac{F_x}{F_y} + \frac{\varepsilon_1}{F_y} = 0$$

take $h \to 0$. $\displaystyle\lim_{h\to 0}\left(\frac{k}{h} + \frac{F_x}{F_y}\right) = 0.$

$$\lim_{h\to 0}\frac{k}{h} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = f'(x) = y' = -\frac{F_x}{F_y}.$$

how about 2nd derivative? $\quad F_x + F_y \cdot y' = 0$

$$dF = F_x\,dx + F_y\,dy$$

$$y'' = \left(-\frac{F_x}{F_y}\right)' = -\frac{F_y(F_{xx} + F_{xy}\cdot y') - F_x(F_{yx} + F_{yy}\cdot y')}{F_y^2}$$

$$= -\frac{F_{xx}F_y^2 - 2F_{xy}F_xF_y + F_{yy}F_x^2}{F_y^3}$$

Exercise:

$$\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} - 1 = 0 \qquad y' = -\frac{F_x}{F_y} = \frac{\beta^2 x}{\alpha^2 y}$$

Review:

metric spaces, complete metric sequence, common handout.