Lec 9/22

Thursday, September 22, 2016 9:03 AM

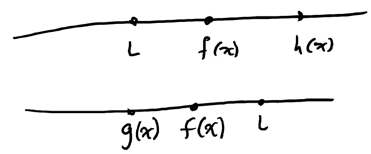
One more result about limits:

Squeeze Theorem for limits:

Suppose $c < a < d$, f, g, h functions satisfying:

1) $g(x) \le f(x) \le h(x) \quad \forall x \in (c,a) \cup (a,d)$

2) $\lim_{x \to a} g(x) = L = \lim_{x \to a} h(x)$



Then $\lim_{x \to a} f(x) = L$.

Proof: Let $\varepsilon > 0$ be given. Observe that $\forall x \in (c,a) \cup (a,d)$,

$|f(x) - L| \le \max\left(|g(x) - L|, |h(x) - L|\right)$. Pick $\delta_1, \delta_2 > 0$

$$0 < |x - a| < \delta_1 \Rightarrow x \in \mathrm{dom}(g) \text{ and } |g(x) - L| < \varepsilon$$

and $$0 < |x - a| < \delta_2 \Rightarrow x \in \mathrm{dom}(h) \text{ and } |h(x) - L| < \varepsilon$$

Let $\delta = \min(\delta_1, \delta_2, a - c, d - a)$

$0 < |x - a| < \delta \Rightarrow x \in (c,a) \cup (a,d) \subseteq \mathrm{dom}(f) \cap \mathrm{dom}(g) \cap \mathrm{dom}(h)$

and $|g(x) - L| < \varepsilon$

and $|h(x) - L| < \varepsilon$

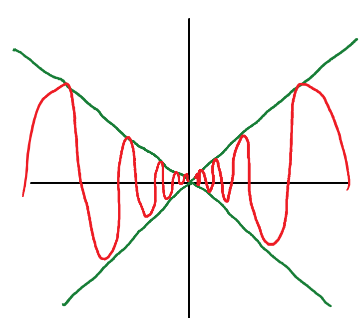
$\Rightarrow x \in \mathrm{dom}(f)$ and

$|f(x) - L| \le \max\left(|g(x) - L|, |h(x) - L|\right) < \varepsilon$ ∎

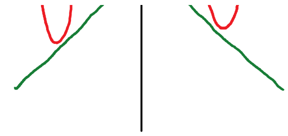
Ex: $\lim_{x \to 0} x \sin\frac{1}{x} = 0$

$-|x| \le x \sin\frac{1}{x} \le |x|$

for $x \in (-\infty, 0) \cup (0, \infty)$



for $x \in$



Note: there are one-sided versions of squeeze thm. too.

Midterm review

1) $$\sum_{i=n^2+1}^{(n+1)^2} i = n^3 + (n+1)^3$$

2) first, $1 < 2 \Rightarrow 1 < \sqrt{2} \Rightarrow 0 < \frac{1}{\sqrt{2}} < 1 \Rightarrow 0 < \frac{s-r}{\sqrt{2}} < s-r$

adding r $\quad r < r + \frac{s-r}{\sqrt{2}} < s$

3) $f(0) + g(y) = 0 \Rightarrow g(y) = -f(0)$ (constant)

$f(x) + g(0) = 0 \Rightarrow f(x) = -g(0)$ (constant)

$$\underbrace{-f(0) - g(0)}_{\text{constant}} = \underbrace{xy}_{\text{not constant}} \quad \forall x, y$$

4) yesterday

5) $\delta = \min\left(1, \frac{\varepsilon}{6(|a|+1)^2}\right)$ forgot to distribute the 5 fully.

also this was the one from the hw

don't have to simplify complicated M expressions

6)