Lec 9/5

Tuesday, September 5, 2017 12:54

Strassen's algorithm: $\Theta(n^{\log_2 7})$

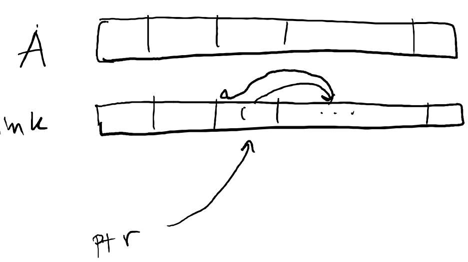
Closest pair problem:

given $A = \{(x_i, y_i) : 1 \le i \le n\} \subseteq \mathbb{R}^2$, find two points whose distance is minimal.

straightforward method $\Theta(n^2)$

Divide-and-conquer: $O(n \log n)$

Linked-list Mergesort:



$A = B \sqcup C$, find closest pair in each; then find pairs btwn B & C

return the closest of the 3.

Desired: $T(n) = 2T(n/2) + O(n) \in O(n \log n)$

Which requires 'divide' part to be $O(n)$

to divide, sort A by x coordinates (do this once $\in \Theta(n \log n)$).

Then $B = A[1,\ldots,\lfloor n/2 \rfloor]$, $C = A[\lfloor n/2 \rfloor + 1, \ldots, n]$

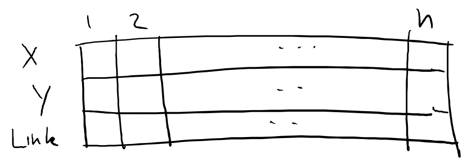
or $A[i,j] = A[i,m] \sqcup A[m+1,j]$ where $m = \lfloor \frac{i+j}{2} \rfloor$

we will write procedure

Closest-pair-between-two-sets $(A[i,j], ptr, \delta, (p_3, q_3))$

which must be $\in O(j-i)$

$A[i] = (X[i], Y[i])$



introduce $A[0] = (-\infty, -\infty)$, $A[n+1] = (\infty, \infty)$ to indicate 'no pair'

Main program:

Global variable $A[0..n+1]$

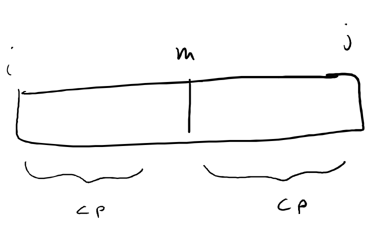
sort $A[1..n]$ s.t. $X[1] \le X[2] \le \ldots \le X[n]$.

Call Closest-pair

Closest-pair:

$\vdots$

$ptr \leftarrow$ mergesort A by y-coord.



sort A[i:j] by y. if mergesort then $T(n) \in O(n \log^2 n)$. bad.

so mergesort is doubled simply implement it alongside CP to avoid this.

$A[1 \ldots \frac{n}{2^p}]$ is sorted by y p+1 times.

Closest-pair-between-two-sets:

only consider points in a δ-box. there are at most 3 such points (because of how δ is defined).