Lec 8/22

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Quiz: 1 True ✓

2 $\theta(2^{4n^2+3n})$ ✓

3 $\Theta(n^5)$

Math foundations

Analysis: runtime := count of basic operations.

$T(n)$:= worst case runtime.

↳ sometimes write $T_w(n)$

for sequential search, $T(n) = 3n+1$ or something $\in O(n)$ or Θ or Ω

$$O(g) = \{ f \in 2^{\mathbb{N}} \mid \exists_{c>0, n_0} \text{ s.t. } f(n) \le c g(n) \ \forall n \geq n_0 \}$$

$\Omega(g) =$ " " $\geq$ " "

$$\Theta(g) = O(g) \cap \Omega(g)$$

Theorem: If $f_1 \in O(g_1)$ and $f_2 \in O(g_2)$ then

$f_1 + f_2 \in O(g_1 + g_2) = O(\max(g_1, g_2))$

$f_1 \cdot f_2 \in O(g_1 g_2)$

Proof: easy constant building.

Same theorem for Ω.

Perhaps better definitions: → using more words and fewer constants

$$O(g) = \{ f \in 2^{\mathbb{N}} \mid \lim_{n\to\infty} \frac{f(n)}{g(n)} \neq \infty \}$$

$$\Omega(g) = \{ \text{"} \qquad\qquad \text{"} \neq 0 \}$$

$$\Theta(g) = O(g) \cap \Omega(g)$$

$$o(g) = \{ f \in 2^{\mathbb{N}} \mid \lim_{n\to\infty} \frac{f(n)}{g(n)} = 0 \}$$

$$\omega(g) = \{ \text{"} \qquad\qquad \text{"} = \infty \}$$

$$\theta(g) = \emptyset$$

If $f \in O(g)$ and $g \in O(h)$ then $f \in O(h)$

Same w/ Θ, Ω.

for $g: \mathbb{N}^k \to \mathbb{N}$, $\quad O(g) = \{ f: \mathbb{N}^k \to \mathbb{N} \mid \exists\, c, r > 0 \text{ s.t. } f(\vec{n}) \leq g(\vec{n}) \text{ when } |\vec{n}| > r \}$

$$4m^2 + 9mn^2 + 6n^3 \notin O(m^2 + n^3)$$

since when $m = n^{1.5}$, $mn^2 \in \omega(m^2 + n^3)$

2) $\sqrt{n}$