Lec 11/9

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Maximum Flow:

Flow network

Directed graph $G = (V, E)$

Source, sink $s, t \in V$.

Capacity $c : V \times V \rightarrow \mathbb{R}$

flow: $f : V \times V \longrightarrow \mathbb{R}$

Capacity constraint: $\forall u, v \in V$, $f(u,v) \leq c(u,v)$

skew symetry: $\forall u, v \in V$, $f(u,v) = -f(v,u)$

flow conservation: $\forall u \in V \setminus \{s,t\}$

$$f(u, V) := \sum_{v \in V} f(u,v) = 0$$

Value of flow is $|f| = f(s,t)$

$$f(X, Y) := \sum_{u \in X} \sum_{v \in Y} f(u,v)$$

$$f(X, X) = 0$$

$f(X, Y) = -f(Y, X)$

$f(X \cup Y, Z) = f(X, Z) + f(Y, Z)$ if $X \cap Y = \emptyset$

$f(X, Y \cap Z) = f(X, Y) + f(X, Z)$ if $Y \cap Z = \emptyset$.

residual network: Network of residual Capacities.

Ford-Fulkerson:

Initially, flow $|f| = 0$.

find augmenting path P, augment flow.

while there is one.

Running time: $O(V + E)$ to find path

$= O(E)$

this will repeat maximally $|f^*|$ times.

so $\in O(E|f^*|)$