Lec 11/16

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Edmonds-Karp : shortest augmenting path.

$\delta(v)$ = shortest distance from s to v.

(u,v) can become critical at most $V/2$ times.

$O(VE^2)$

Push-relabel Algorithm $O(V^2E)$.

<u>Preflow</u>: "relaxed" flow conservation.

$\forall u \in V \setminus \{s\}, \quad f(V,u) \geq 0$

$e(u) = f(V,u) =$ excess flow.

if $e(u) > 0$, u is overflowing.

height function: $h : V \longrightarrow \mathbb{N}_0$

residual.

$h(s) = |V|, \quad h(t) = 0, \quad h(u) \leq h(v) + 1 \quad \forall (u,v) \in E_f$

Push(u,v) operation:

Applicable when u is overflowing, $c_f(u,v) > 0$, $h(u) = h(v) + 1$

Push $\min\{e(u), c_f(u,v)\}$ units of flow from u to v.

Saturating vs Nonsaturating push

Initial preflow:

$$f(u,v) = \begin{cases} C(u,v) & \text{if } u = s \\ -C(u,v) & \text{if } v = s \\ 0 & \text{o.w.} \end{cases}$$

$$e(v) = \begin{cases} C(s,v) & \text{if } (s,v) \in E \\ -\sum \{C(s,x) : (s,x) \in E\} & \text{if } s = v \\ 0 & \text{o.w} \end{cases}$$

$$h(v) = \begin{cases} |V| & \text{if } s = v \\ 0 & \text{o.w.} \end{cases}$$