Lec 4/16

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$R_{e,B}: \chi_A \neq \Phi_e^B$

$R_{e,C}: \chi_A \neq \Phi_e^C$

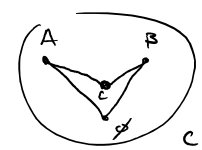
Lemma 2: let $m = \lim_s L(1,B,s)$. Then at least one $y \leq m$ is a permanent witness for $R_{1,B}$, i.e. $A(y) \neq \Phi_1^B(y)$.

Pf Let s be the first stage at which m is hit.
If $\exists y < m$ that enters A after stage s, then y is a permanent witness since $\forall t \geq s, [\Phi_1^B(y)[t]\downarrow = 0]$.
So assume that no $y < m$ enters A after stage s.
Then m is the permanent witness since m is the high-water mark, So if it were not a permanent witness we'd get a contradiction. □

We argue by induction on the priority that each requirement acts only finitely often.

New theorem

DFN: A pair of nonrecursive sets A and B is minimal if $(\forall C)[(C \leq_T A \ \& \ C \leq_T B) \Longrightarrow C$ is recursive$]$.



This pair is not possible, $C \equiv \emptyset$.

Lachlan-Yates Thm: There is a minimal pair of r.e. sets.

Proof: We'll construct A and B in stages to satisfy the following requirements:

$R_e: \overline{A} \neq W_e$, $Q_e: \overline{B} \neq W_e$, $N_e: (\Phi_e^A = \Phi_e^B =$ some total fn $f) \Rightarrow f$ is computable.