Lec 3/30

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R_{2e+1} : $\chi_A \neq \Phi_e^B$

R_{2e+2} : $\chi_B \neq \Phi_e^A$

Stage 0: Define $\alpha_0 = \beta_0 = \Lambda$

Stage 2e+1: ~

Stage 2e+2: We define α_{2e+2} and β_{2e+2} to satisfy R_{2e+2} as follows:

Let $n = |\beta_{2e+1}| + 1$.

If $(\exists \sigma)(\exists t)[\alpha_{2e+1} \subset \sigma$ and $\Phi_{e,t}^{\sigma}(n)\downarrow]$ then

define $\alpha_{2e+2} = \sigma$, $\beta_{2e+2} = \beta_{2e+1} \wedge 0$.

otherwise define $\alpha_{2e+2} = \alpha_{2e+1} \wedge 0$ (or 1), $\beta_{2e+2} = \beta_{2e+1} \wedge 1$.

Can you do this w/ r.e. languages?

yes

lemma: $(\forall A)[A$ is r.e. $\Rightarrow A \leq_T K]$. The converse is false.

Friedberg-Muchnik Theorem: $\exists$ incomparable r.e. languages.

Pf "finite injury priority argument" We'll construct incomparable r.e. sets A, B in stages.

Ie we'll define sets $\overbrace{A_1 \subseteq A_2 \subseteq \cdots}^{\text{all finite}}$ and $A = \bigcup_{s=1}^{\infty} A_s$

and $B_1 \subseteq B_2 \subseteq \cdots$ and $B = \bigcup_{s=1}^{\infty} B_s$.

Requirements are the same: R_{2e+1} : $\chi_A \neq \Phi_e^B$ and R_{2e+2} : $\chi_B \neq \Phi_e^A$.

The strategy for meeting a single requirement R_{2e+1} is to attach to it a potential witness x not yet in A.

If a stage s reached s.t. $\Phi_{e,s}^{B_s}(x)\downarrow=0$ then we say R_{2e+1} "needs attention" at stage $s+1$. Analogously, R_{2e+2} needs attention at stage $s+1$ if $\Phi_{e,s}^{A_s}(x)\downarrow=0$. We attend to the one of strongest priority (i.e. of smallest index).

Assume the strongest priority requirement that needs attn is $j=2e+1$.

then we do

(1) $A_{s+1} \leftarrow A_s \cup \{x\}$

(2) update the restraint function:

$r_j^{s+1} \longleftarrow 1 +$ the rightmost position of the oracle head during the computation of $\Phi_{e,s}^{B_s}(x)$