Lec 3/21

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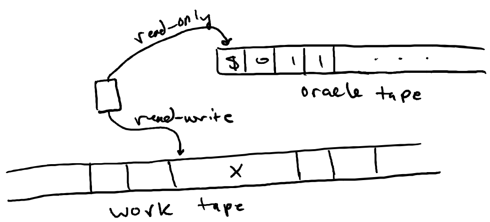
$A \leq_m B$ f computable

$A \leq_p B$ f computable in poly-time

$A \leq_T B$ "oracle"

Oracle Turing Machine



Characteristic f_n for $B \subseteq \omega$:

$$\chi_B(n) = \mathbb{1}_B(n) = \begin{cases} 1 & n \in B \\ 0 & n \notin B \end{cases}$$

Transition function:

$$\delta : Q \times \Gamma \times \{\$, 0, 1\} \longrightarrow Q \times \Gamma \times \{L, R\} \times \{L, R\}.$$

Let M_e^B denote the e^{th} oracle TM with oracle B, and let φ_e^B denote the partial function that it computes.

If $\overset{\text{for some } e}{M_e^B}$ halts on all inputs and $\overbrace{A = \{n : \varphi_e^B(n) = 1\}}^{\varphi_e^B = \chi_A}$ Then $A \leq_T B$.

"A turing reduces to B"

Simple properties of $\leq_T$:

(i) $A \leq_T A$.

(ii) $A \leq_T B$ and $B \leq_T C \implies A \leq_T C$.

(iii) $A \leq_T B$ and B is recursive $\implies A$ is recursive.

(iv) $A \leq_m B \implies A \leq_T B$ (converse is ~~probably~~ false).

try 1

$L = \{e : M_e(e)\downarrow \text{ and } M_{e+1}(e)\downarrow\}$

$L \leq K$

(iii) $A \leq_T B$ and B is recursive $\Longrightarrow$ A is recursive.

$$L = \{e : M_e(e)\downarrow \text{ and } M_{e+1}(e)\downarrow\}$$
$$L \leq_T K \ \ldots$$

(iv) $A \leq_m B \Longrightarrow A \leq_T B$ (converse is ~~probably~~ false).

$\overline{K} \leq_T K$ but $\overline{K} \not\leq_m K$ since $\overline{K}$ is not r.e.

(v) $A \leq_T \overline{A}$

(vi) A is r.e. $\Rightarrow A \leq_T K$ (K is r.e.-complete under $\leq_T$).

in fact $A \leq_m H$. so $A \leq_m K$.

Pf: let $A = L(M_{e_0})$. $A \leq_T H$ since given x we can check if $M_{e_0}(x)$ runs forever. If so, we reject, if not we run $M_{e_0}(x)$, which is guaranteed to halt. We've already seen that $H \leq_m K$ so $H \leq_T K$ so $A \leq_T K$.

$A \leq_T B \not\Rightarrow B \leq_T A$. let $A = \{17\}$, $B = K$. $A \leq_T B$ but if $K \leq_T \{17\}$ then K would be recursive, not true.