

Subset Sum :

Instance: positive #s a_1, a_2, \dots, a_r and t .

Question: Are there i_1, i_2, \dots, i_k s.t. $\sum_{j=1}^k a_{i_j} = t$

Obviously in NP. Also NP-complete by the following reduction:

3SAT \leq_P SS

$E = C_1 \wedge \dots \wedge C_m$
 over vars x_1, \dots, x_n

$C_1 = x_1 \vee \bar{x}_2 \vee x_n$
 $C_2 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$

fillers $\left\{ \begin{array}{l} y_1 \\ z_1 \\ \vdots \\ y_n \\ z_n \\ t \end{array} \right.$

	1	2	3	...	n	C_1	C_2	...	C_m
x_1	1	0	0	...	0	1			
\bar{x}_1	1	0	0	...	0	0			
x_2		1	0	...	0	0			
\bar{x}_2		1	0	...	0	1			
\vdots									
x_n						1			
\bar{x}_n						1			
y_1							1	0	0
z_1							1	0	0
\vdots								1	0
y_n								1	0
z_n								1	0
t	1	1	1	...	1	3	3	...	3

Partition:

Instance: Positive integers a_1, a_2, \dots, a_n .

Question. is there a subsequence $a_{i_1}, a_{i_2}, \dots, a_{i_k}$

$$\text{s.t. } \sum_{j=1}^k a_{i_j} = \frac{1}{2} \sum_{i=1}^n a_i .$$

Clearly $\in NP$. NP complete

Since Subset Sum \leq_p Partition

Subset sum

$$a_1, a_2, \dots, a_r \Rightarrow$$

t