Lec 3/19

Monday, March 19, 2018 12:42

Set Covering Optimization Problem.

Input: A finite set X and a family F of subsets of X.

Output: A Minimum-Size cover of X.

Algorithm:

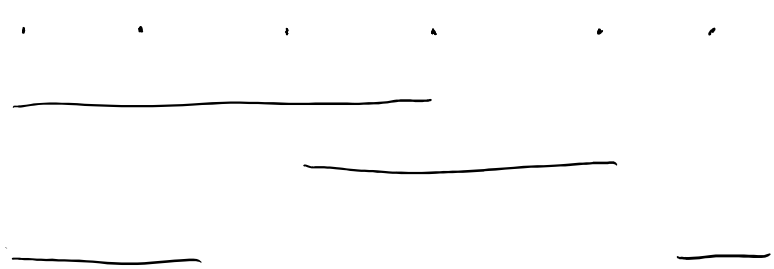
$i \leftarrow 0$

$U_0 \leftarrow X$ /* U_i is the set of uncovered elements after iteration i */

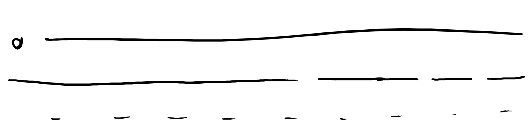
<u>while</u> $U_i \neq \emptyset$ do

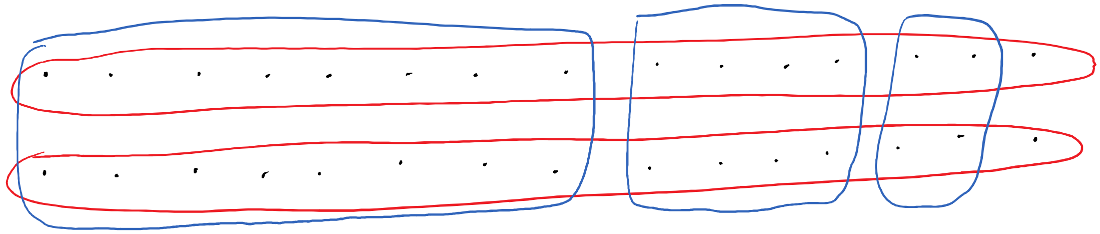
- select some $T_{i+1} \in F$ that maximizes $|T_{i+1} \cap U_i|$
- $U_{i+1} \leftarrow U_i - T_{i+1}$
- $i \leftarrow i+1$

return $\{T_1, T_2, \ldots, T_i\}$



gives suboptimal result.





optimal greedy $\frac{\text{GREEDY}}{\text{OPTIMAL}} \approx \frac{\log_2(n)}{2} \longrightarrow \infty.$

Fix some instance $\langle X, F \subseteq \text{POW}(X) \rangle$ of Set covering and let $n = |X|$ and let OPT = the size of an optimal solution for this instance.

<u>Theorem</u>: $\frac{\text{GRE}}{\text{OPT}} \in O(\log(n))$.

<u>Lemma</u>: $|U_i| \leq n\left(1 - \frac{1}{\text{opt}}\right)^i$ for all $i \geq 0$.

<u>proof</u>: Base case $|U_0| = n$

inductive: Assume $|U_i| \leq n\left(1 - \frac{1}{\text{opt}}\right)^i$. Since X can be covered w/ opt sets, so can U_i. So there is a set $T \in F$ s.t. $|T \cap U_i| \geq \frac{|U_i|}{\text{OPT}}$

So Greedy algorithm makes a choice at least this big so:

$$|U_{i+1}| \leq |U_i| - |T| \leq n\left(1 - \frac{1}{\text{OPT}}\right)^i - \frac{n\left(1 - \frac{1}{\text{opt}}\right)^i}{\text{opt}}$$

$$= n\left(1 - \frac{1}{\text{OPT}}\right)^i\left(1 - \frac{1}{\text{opt}}\right) = n\left(1 - \frac{1}{\text{OPT}}\right)^{i+1}. \quad \square$$

<u>Pf of Theorem</u>: Let $b = 1 - \frac{1}{\text{OPT}}$ and let x be the real # s.t. $n b^x = 1$

so $n = \left(\frac{1}{b}\right)^x$ so $\ln(n) = -x \ln(b) \Rightarrow x = -\frac{\ln(n)}{\ln(b)}$. $\square$