Lec 2/5

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Def: A <u>Partial function</u> f is a subset of $\omega \times \omega$ so that if (a,b), $(a,c) \in f$ then $b=c$. If $(n,a) \in f$ we say $f(n)=a$.

Thus a total function is a special case of a partial function.

If f and g are partial functions then $f=g$ iff $\forall n \left[f(n)=g(n) \Leftrightarrow \text{either both are undefined or both are defined \& equal} \right]$

Def A partial function is called <u>partial recursive</u> if it is computed by some TM. That is, $\exists M$ s.t. $\forall n$, if $f(n)$ is defined, then $M(n)\downarrow$ with value $f(n)$ as its output. otherwise, $M(n)\uparrow$ (may or may not halt).

Note: a computable fn is partial recursive, & a p.r.f. defined $\forall n \in \omega$ is cptble.

We denote partial function computed by TM M_i by φ_i.

<u>S_{mn} theorem</u> Let f be a p.r.f. then $\exists$ a total recursive function σ s.t. $\forall i,j$, $\varphi_{\sigma(i)}(j) = f(\langle i,j \rangle)$

<u>Proof</u> given i, let M be a turing machine that, given input j, encodes $\langle i,j\rangle$ on the tape & then simulates the machine which computes f on this input. Let $\sigma(i)$ be the index of M. □

This is computable, intuitively.

<u>Recursion theorem (fixed-point form)</u>: Let σ be a total recursive function. Then $\exists i$ s.t. $\varphi_i = \varphi_{\sigma(i)}$. (there is a fixed point of $\lambda x.\ \varphi_{\sigma(x)}$).

<u>Proof.</u> Consider the p.r.f. $f(\langle i,j\rangle) = \varphi_{\sigma(\varphi_i(i))}(j)$. This is p.r. since every action necessary to compute it is partial recursive. (if $\varphi_i(i)$ is undefined then $f(\langle i,j\rangle)$ is too). By S_{mn} theorem,

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"rabbit_out_of_the_hat proof"

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$\sigma(\varphi_i(i))$. Let $n \in \omega$ s.t. M_n compute g. As a special case of (*), $\varphi_{g(n)} = \varphi_{\sigma(\varphi_n(n))}$. But $\varphi_n = g$ Since M_n computes g. So $\varphi_{g(n)} = \varphi_{\sigma(g(n))}$. Thus the theorem is proved ($g(n)$, the desired fixed pt exists since g is total) $\square$