Lec 2/26

Monday, February 26, 2018 12:35

Set covering

Instance: A family $F = \{S_1, S_2, \ldots, S_n\}$ of finite sets and an integer $K \in \{1, \ldots, n\}$.

Question: Are there indices $i_1, i_2, \ldots, i_k$ s.t. $\bigcup_{j=1}^{k} S_{i_j} = \bigcup_{i=1}^{n} S_i$

This problem is NP-complete.

- it's in NP, interpret guess as $\{i_1, i_2, \ldots, i_k\}$.
- $VC \leq_p SC$ by edges $\longrightarrow$ elements

  vertices $\longrightarrow$ sets, containing edges incident on vertex.

VC w/ $K = 100$

instance: Undirected graph $G = (V, E)$

Question: does G have a v.c. of size 100.

This is in P because we could examine each of the $\binom{n}{100}$ subsets of V.

$$\frac{n!}{(n-100)! \cdot 100!} = \frac{1}{100!}(n)(n-1)(n-2)\cdots(n-99)$$

VC w/ $k = \sqrt{n}$

instance: $G = (V, E)$

instance: $G=(V,E)$

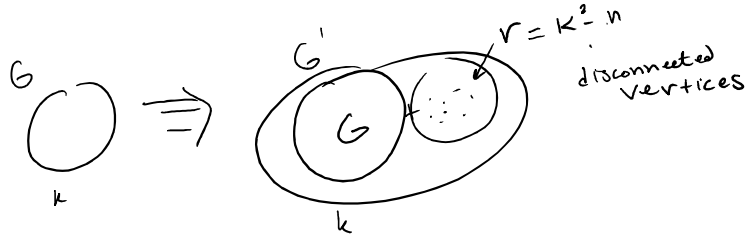
Question: does V.C. have a V.C. of size $\lceil\sqrt{n}\rceil$?

$\binom{n}{\sqrt{n}}$ is not polynomial in n.

reduce VC to this:

Case 1: $k=\lceil\sqrt{n}\rceil$

Case 2: $k>\lceil\sqrt{n}\rceil$



Case 3: $k<\sqrt{n}$