Lec 2/23

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3SAT

<u>Instance</u>: Collection $C = \{c_1, \ldots, c_m\}$ of clauses over a set $X = \{x_1, \ldots, x_n\}$ s.t. each clause has exactly 3 literals, $|c_j| = 3 \ \forall j$.

<u>Question</u>: is there a truth assignment $\tau : X \to \{T, F\}$ that satisfies all clauses?

Thus 3SAT is a subproblem of SAT.

<u>Lemma</u>: if A is a subproblem of $B \in NP$, $A \in NP$.

Want to show SAT $\leq_p$ 3SAT so that 3SAT is NP-complete.

$$E \Longrightarrow E'$$

Replace a clause like $\overbrace{(a \vee b \vee c \vee d \vee e \vee f \vee g)}^{\text{long clause, more than 3.}}$ in E by

$$(a \vee b \vee y_1) \wedge (\bar{y}_1 \vee c \vee y_2) \wedge (\bar{y}_2 \vee d \vee y_3) \wedge (\bar{y}_3 \vee e \vee y_4) \wedge (\bar{y}_4 \vee f \vee g)$$

where y_1, y_2, y_3, y_4 appear nowhere else in E'.

Claim: E satisfiable iff E' is.

Replace a clause like (a) by $(a \vee y_1 \vee y_2) \wedge (a \vee \bar{y}_1 \vee y_2) \wedge (a \vee y_1 \vee \bar{y}_2) \wedge (a \vee \bar{y}_1 \vee \bar{y}_2)$.

Replace a clause like $(a \vee b)$ by $(a \vee b \vee y_1) \wedge (a \vee b \vee \bar{y}_1)$.

So there you have it.

<u>Vertex Cover Problem</u> (VC):

Defn A vertex cover of an undirected graph $G=(V,E)$ is a subset $V' \subseteq V$ s.t. $(u,v) \in E$ only if $u \in V'$ or $v \in V'$.

<u>Instance</u>: An undirected graph $G=(V,E)$ and an integer K.

<u>Question</u>: Does G have a vertex covr of size K.

<u>Claim</u>: VC is NP-complete.

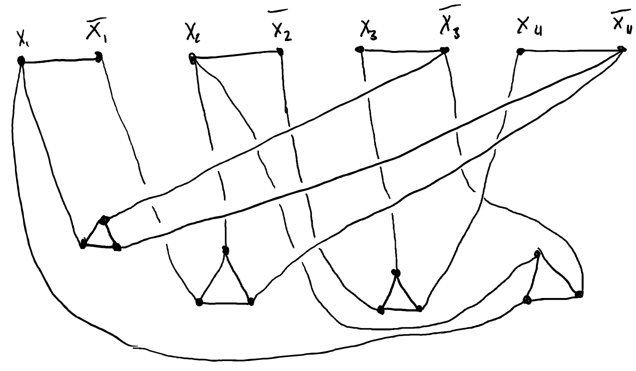
<u>Pf</u> $VC \in NP$ since we can interpret the guess as the vertex cover (bit $-i$ will be 0 if $i \notin V'$, 1 if $i \in V'$).

And $3SAT \leq_p VC$.

3SAT $\Longrightarrow$ VC

$(x_1 \vee \bar{x}_3 \vee \bar{x}_4)(\bar{x}_1 \vee x_2 \vee \bar{x}_4)$
$(\bar{x}_2 \vee x_3 \vee x_4)(x_1 \vee x_2 \vee \bar{x}_3)$

m clauses over
n variables



$K = 2m + n$

<u>Claim</u> E is satisfiable iff G has a vc of size k.

$\Rightarrow$: let τ satisfy E

$\Leftarrow$: let V' be a k' cover of G.