Continuation of pf of Cook-Levin Theorem.

Variables of E:

Variable	range	intended meaning
Q_k^i	$0 \le k \le r$ $0 \le i \le p(n)$	true iff M is in state q_k at time i (having been started on X).
H_j^i	$-p(n) \le j \le p(n)$ $0 \le i \le p(n)$	the read-write head of M is scanning square j at time i.
$S_{j,\alpha}^i$	$0 \le i \le p(n)$ $-p(n) \le j \le p(n)$ $\alpha \in \{0,1,B\}$	Symbol α is in square j at time i.

Clauses of E:

	Clauses	range	intended meaning
group 1 $p(n)+p(n)\binom{r}{2}$ total clauses here	$(Q_0^i \wedge Q_1^i \wedge \cdots \wedge Q_r^i)$ $(\overline{Q_k^i} \vee \overline{Q_{k'}^i})$ $(\forall k,k')$	$0 \le i \le p(n)$ $0 \le k < k' \le r$	M is in exactly one state at time i.
group 2: $p(n)+p(n)\binom{2p(n)}{2}$	$(H_{-p(n)}^i \wedge H_{-p(n)+1}^i \wedge \cdots \wedge H_{p(n)}^i)$ $(\overline{H_j^i} \vee \overline{H_{j'}^i})$	$0 \le i \le p(n)$ $-p(n) \le j < j' \le p(n)$	R/W head of M over exactly one square at time i.
group 3: $6p(n)^2 + 3p(n)\binom{2p(n)}{2}$	$(S_{j,0}^i \vee S_{j,1}^i \vee S_{j,B}^i)$ $(\overline{S_{j,\alpha}^i} \vee \overline{S_{j,\alpha'}^i})$	$0 \le i \le p(n)$ $-p(n) \le j \le p(n)$ $\alpha \in \{0,1,B\}$	every square has exactly one symbol at time i.

group4: $p(n)+3$

$$\begin{cases} Q_0^0,\ H_0^0,\ S_{0,B}^0 \\ S_{1,x_1}^0,\ S_{2,x_2}^0, \dots, S_{n,x_n}^0 \\ S_{n+1,B}^0, \dots, S_{p(n),B}^0 \end{cases}$$

$S_{j,B}^0 \Rightarrow S_{j-1,B}^0$

$-p(n) \le j \le -1$

initial configuration

group5: $\left(Q_1^1 \vee Q_1^2 \vee \dots \vee Q_1^{p(n)} \right)$

M accepts x within time $p(n)$.

group 6:

$$\left(\overline{H_j^i} \wedge S_{j,\alpha}^i \right) \Rightarrow S_{j,\alpha}^{i+1}$$

$0 \le i \le p(n)$
$-p(n) \le j \le p(n)$
$\alpha \in \{0,1,B\}$

tape won't change if r/w head isn't scanning there.

note: $(\overline{A} \wedge B) \Rightarrow C \iff (A \vee \overline{B} \vee C)$

$$\left(H_j^i \wedge S_{j,\alpha}^i \wedge Q_k^i \right) \Rightarrow \left(H_{j+\delta}^{i+1} \wedge S_{j,\alpha'}^{i+1} \wedge Q_{k'}^{i+1} \right)$$

subset of

$0 \le i \le p(n)$ $k, k' \in \{0,1,\dots,r\}$
$-p(n) \le j \le p(n)$
$\alpha \in \{0,1,B\}$
$\alpha' \in \{0,1,B\}$

transition function

$\overset{-1}{L}$ or $\overset{+1}{R}$

Where $\delta_M(\alpha, q_k) = (\delta, \alpha', q_{k'})$

note $(A \wedge B \wedge C) \Rightarrow (D \wedge E \wedge F) \iff (\underbrace{\overline{A} \vee \overline{B} \vee \overline{C}}_{X} \vee (D \wedge E \wedge F))$

$\iff (X \vee D) \wedge (X \vee E) \wedge (X \vee F)$

<u>Claim</u>: E is satisfiable iff M accepts x.