

$$\forall e, \exists \text{inf many } i \text{ s.t. } W_i = W_e.$$

Does  $R$  have an infinite recursive subset?

Yes, let  $A = \{e : M_e(e) \text{ enters the rejecting state within 7 steps}\}$

First,  $A \in R$  trivial.

Next  $A$  infinite also trivial ( $\exists$  inf. many TMs which reject on 1st step).

And  $A$  is recursive bc we just simulate  $M_e(e)$  for 7 steps.

Padding Argument.

Let  $L$  be an undecidable language.

let  $A = \{i \in L : i \text{ is even}\}$ ,  $B = \{i \in L : i \text{ is odd}\}$ .

i) Must at least one of these be decidable?

ii) " " undecidable?  $\rightarrow$  yes, clearly.

No: consider  $L = \{2i : i \in K\} \cup \{2i+1 : i \in H\}$

or  $L = \{2i, 2i+1 : i \in K\}$ .

so  $A = 2K$ ,  $B = 2K+1$ ,  $L$  is undecidable

Since  $K \leq_n L$ , and  $K \leq_m A$ ,  $K \leq_m B$

$\uparrow$	$\uparrow$	$\uparrow$
$f(x) = 2x$	$f(x) = 2x$	$f(x) = 2x+1$

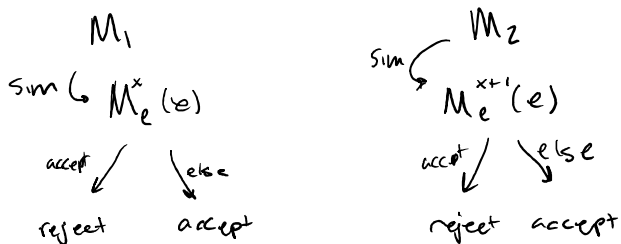
Let  $EQ = \{\langle e_1, e_2 \rangle : W_{e_1} = W_{e_2}\}$

$R \leq_m EQ$

Let  $EQ = \{ \langle e_1, e_2 \rangle : W_{e_1} = W_{e_2} \}$

$\bar{K} \leq_m EQ$

So  $EQ$  Not r.e.



$f(e) = \langle \text{index } M_1, \text{index } M_2 \rangle$ . if  $e \in \bar{K}$ ,  $M_e^x(e)$  always rejects, so  $L(M_1) = \omega = L(M_2)$  so  $f(e) \in EQ$ .

else  $\exists x$  s.t.  $M_e^x(e) \uparrow$ ,  $M_e^{x+1}(e) \downarrow$  so

$L(M_1) = \{1, 2, \dots, x\}$ ,  $L(M_2) = \{1, 2, \dots, x-1\}$ ,

so  $f(e) \notin EQ$ .

Another proof:  $EMPTY \leq_m EQ$ .

Let  $z$  be the index of a TM which goes to its rejecting state on the first step. Then  $\forall e, e \in EMPTY$  iff  $\langle e, z \rangle \in EQ$ .

### Recursion Theorem

Consequences: ① Number theory, ②  $\exists e : W_e = W_{f(e)}$   $\forall$  computable  $f$ .

③  $\exists e (M_e \text{ halts w/ output } e)$