Lec 2/2

Friday, February 2, 2018 12:39

$\forall e, \exists$ inf many $\hat{e}$ s.t $W_{\hat{e}} = W_e$.

Does $\overline{K}$ have an infinite recursive subset?

Yes, let $A = \{ e : M_e(e)$ enters the rejecting state within 7 steps $\}$

First, $A \subseteq \overline{K}$ trivial.

Padding Argument.

Next A infinite also trivial $\underbrace{(\exists \text{ inf. many TMs which reject on 1st step})}$.

And A is recursive bc we just simulate $M_e(e)$ for 7 steps.

Let L be an undecidable language.

Let $A = \{ i \in L : i \text{ is even} \}$, $B = \{ i \in L : i \text{ is odd} \}$.

i) Must at least one of these be decidable?

ii) " " undecidable? $\longrightarrow$ Yes, clearly.

$\rightarrow$ No: consider $L = \{ 2i : i \in K \} \cup \{ 2i+1 : i \in H \}$

or $L = \{ 2i, 2i+1 : i \in K \}$.

so $A = 2K$, $B = 2K+1$, L is undecidable

Since $K \leq_m L$, and $K \leq_m A$, $K \leq_m B$

$f(x) = 2x$ $\quad$ $f(x) = 2x$ $\quad$ $f(x) = 2x+1$

Let $EQ = \{ \langle e_1, e_2 \rangle : W_{e_1} = W_{e_2} \}$

$\overline{K} \leq_m EQ$

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So EQ Not r.e.

M_1: sim $M_e^x(e)$; accept $\to$ reject, else $\to$ accept

M_2: sim $M_e^{x+1}(e)$; accept $\to$ reject, else $\to$ accept

$f(e) = \langle \text{index } M_1, \text{ index } M_2\rangle$. if $e \in \overline{K}$, $M_e^x(e)$ always reject, so $L(M_1) = \omega = L(M_2)$ so $f(e) \in EQ$.

else $\exists\, x$ s.t. $M_e^x(e)\uparrow$, $M_e^{x+1}(e)\downarrow$ so

$L(M_1) = \{1, 2, \ldots, x\}$, $L(M_2) = \{1, 2, \ldots, x-1\}$,

So $f(e) \notin EQ$.

Another proof: $EMPTY \leq_m EQ$.

Let z be the index of a TM which goes to its rejecting state on the first step. Then $\forall e$, $e \in EMPTY$ iff $\langle e, z\rangle \in EQ$.

Recursion Theorem

Consequences: ① Number theory, ② $\exists e : W_e = W_{f(e)}$ $\forall$ computable f.

③ $\exists e$ (M_e halts w/ output e)