Lec 2/19

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Lemmas: ($L_2 \in NP$, L_1 is NP-complete, $L_1 \leq_p L_2$) $\Rightarrow$ L_2 is NP-complete

Pf: $\leq_p$ is transitive.

P is closed under complementation

NP might be...

Give an example of a problem B s.t.

i) $B \notin NP$

ii) $\forall L \in NP$, $L \leq_p B$.

Let A be an NP-complete problem.

Let $B = 2A \cup (2K+1)$. $K \leq_m B$, so $B \notin NP$.

$\forall L \in NP$, $L \leq_m B$ by $2f$ where f reduces L to A.

Satisfiability (SAT):

Instance: a Boolean expression E in conjunctive normal form over the variables $x_1, x_2, \dots, x_n$

Question: is E satisfiable. i.e. is there a truth assignment $\tau \in \{T,F\}^n = \{T,F\}^{\{x_1,\dots,x_n\}}$ that makes each clause in E evaluate to T.

conjunctive normal form:

variables: $x_1, x_2, \dots, x_n$ which take values T or F

CNF

<u>Variables</u>: $x_1, x_2, \ldots, x_n$ which take values T or F

<u>Literals</u>: $x_1, \bar{x}_1, x_2, \bar{x}_2, \ldots, x_n, \bar{x}_n$.

<u>Clauses</u>: disjunction of literals (i.e. $x_5 \vee \overline{x_{16}} \vee \overline{x_{30}} \vee x_{47}$)

<u>expressions</u>: conjunction of clauses (i.e. $(x_5 \vee \bar{x}_{16}) \wedge x_2 \wedge (\overline{x_{30}} \vee x_{47})$).

Note: can just juxtapose instead of writing $\wedge$.

<u>Cook-Levin Theorem</u>: SAT is NP-complete.

<u>Proof</u>: Clearly SAT $\in$ NP (interpret guess as truth values).

Let $L \in NP$. We'll show $L \leq_P$ SAT.

Let M be a poly-time NDTM that accepts L. Let P be a polynomial s.t. $T_m(n) \leq P(n)$ $\forall n$. Let $x \in \{0,1\}^*$ and let $n = |x|$. Assume WLOG that $\{q_0, q_1, \ldots, q_r\}$ are the states of M, with q_0 being the start state and q_1 the accepting state.

We must efficiently construct a Boolean expression E in CNF that is satisfiable iff M accepts x.

to be continued...