Lec 2/14

Wednesday, February 14, 2018 12:51

1: A decision problem Π is an ordered pair (D_π, Y_π) where D_π is some countable set and $Y_\pi \subseteq D_\pi$.

eg: Problem of divisibility by 5: $\Pi = (\mathbb{Z}, 5\mathbb{Z})$.

2: A Language is a subset of $\{0,1\}^*$.

3: An encoding scheme e for a decision problem π is a 1-1 function from D_π to $\{0,1\}^*$. The image $e(D_\pi)$ is denoted by $L_{\pi,e}$.

An encoding scheme is reasonable if its integers are encoded in binary. (Hazy Defn).

4: A Deterministic Turing Machine (DTM) has a finite-state control, a read-write head, a two-way infinite tape, each of whose squares may contain 0, 1, or B (blank). Among its states are q_0, q_{acc}, q_{rej}.

5: A DTM M accepts a string $x \in \{0,1\}^*$ if it eventually reaches state q_{acc} having started in q_0 w/ input x.

6: If M is a DTM, $L(M) = \{x \in \{0,1\}^* : M \text{ accepts } x\}$.

7: A DTM M solves a decision problem π under encoding scheme e if M halts on all inputs (i.e. all binary strings) and $L(M) = e(Y_\pi)$.

8: The time used in the computation of a DTM M on input x is the number of steps in that computation before a halting state is reached.

9: Let M be a DTM that halts on all inputs. The time complexity of M is $T_M : \omega \longrightarrow \omega$ defined by $T_M(n) = \max_{x \in \{0,1\}^n} (\text{time used by } M \text{ on } x)$.

10: M is a polynomial-time DTM if $\exists\, p(X) \in \mathbb{N}[X]$ s.t. $\forall n \geq 0$, $T_M(n) \leq p(n)$.

11: $P = \{L(M) : M \text{ is a polynomial-time DTM}\}$.

12: A Nondeterministic Turing Machine (NDTM) is a DTM but with two additional features: a "guessing module" and a "guessing head"

guessing module "guesses" a guess string on the left side of the tape.

13: An NDTM accepts a binary string if there is some guess $y \in \{0,1\}^*$ s.t. if M guesses y with input x, M eventually reaches the accept state.