Lec 1/31

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Let $CF = \{e : W_e \text{ is context-free}\}$

Claim: CF is not r.e.

Proof: We show $\overline{K} \leq_m CF$.

Consider a TM M that does the following on input x:

Simulate M_e on e. If $M_e(e)\downarrow$, then accept if $x \in H$. || if $M_x(x)\downarrow$ then accept else reject

else reject

Let $f(e)$ be index of M. If $e \in \overline{K}$, $M_e(e)\uparrow$, so we reject all x, so $W_{f(e)} = \emptyset$. else, $W_{f(e)} = H$ so

$\nearrow$ ork

so $f(e) \in CF$ so $f(e) \notin CF$

$\square$

Let $A = \{2x : x \in K\} \cup \{2x+1 : x \in \overline{K}\}$.

Claim: $A, \overline{A}$ both not r.e.

Since $\overline{K} \leq_m A$ by $f(x) = 2x+1$,

$K \leq_m A$ by $f(x) = 2x$

so $\overline{K} \leq_m \overline{A}$ by $f(x) = 2x$ as well.

Does K have a non-r.e. subset?

Soln 1: Yes, counting arg. $|\underbrace{P(K)}_{\text{K must be infinite}}| = |\mathbb{R}|$, $|\text{r.e.}| = |\mathbb{N}|$.

Soln 2: Let $\alpha_1, \alpha_2, \ldots$ be the elements of K in the order in which they were printed by an enumerator.

Let $T = \{\alpha_i : i \in \overline{K}\}$.

Claim: T is not r.e. since $\overline{K} \leq_m T$.

~~pf~~ Let $f(e) = \alpha_e$. then $e \in \overline{K}$ iff $f(e) = \alpha_e \in T$ $\square$

Does K (or any r.e. set.) have an infinite recursive subset?

Yes, define $\alpha_1, \alpha_2, \ldots$ as earlier.

Keep track of "biggest seen so far"

$$A = \{\alpha_i : \alpha_i > \alpha_j \text{ for } j < i\} \subset K$$

then the enumerator for K spits these boys out in order, so when you see a bigger # comin out of K, you know your # isn't there.

Also, A is infinite (clearly).

What about $\overline{K}$? does $\overline{K}$ have an infinite recursive subset? Hint: yes.