Lec 1/29

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Let EMPTY $= \{e : W_e = \emptyset\}$ (indices of TMs which accept $\emptyset$).

is EMPTY recursive?

is it r.e.?

Claim1: $\overline{\text{EMPTY}}$ is R.e. (just simulate on all inputs in dovetailing fashion).

Claim3: EMPTY is not r.e. (using fact that it's not recursive).

Recall $K = \{e : M_e(e)\downarrow\}$.

claim2: $K \leq_m \overline{\text{EMPTY}}$. given $e \in \omega$, construct a TM M that does the followin on x: Simulate $M_e(e)$, if it accepts then reject x, else accept x. (DOESNT WORK)

Simulate $M_e^x(e)$, if it accepts then accept x

else reject x

Let $f(e)$ be the index of M. and (thus f is turing computable) if $e \in K$ then $W_{f(e)} = \emptyset$ so $f(e) \notin$ EMPTY & if $e \notin K$ then $f(e) \in$ EMPTY.

$e \in K \implies \exists x\, [M_e^x(e)\downarrow] \implies$ M accepts something $\implies f(e) \notin$ EMPTY

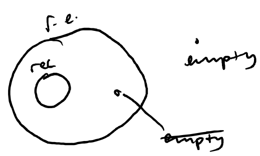
$e \notin K \implies \forall x\, [M_e^x(e)\uparrow] \implies$ M doesn't accept $\implies f(e) \in$ EMPTY

f is clearly turing cptble.

So since $K \le_m \overline{EMPTY}$, $\overline{EMPTY}$ is not recursive

so EMPTY is not recursive & so since

$\overline{Empty}$ is r.e., Empty is not r.e.



In fact, it's easier: Just use $M_e(e)\downarrow$ instead of $M_e^*(e)\downarrow$.

to get same result in Claim 2.

Let $L_{12} = \{e : |W_e| = 12\}$

Is L_{12} recursive?

Is L_{12} r.e.? $L_{\geq 12}$ is r.e. (dovetailing)

Is $\overline{L}_{12}$ r.e.?

Claim: $K \le_m \overline{L}_{12} = L_{\geq 12}$

Proof: given e, construct a TM M that does (given input x):

If $x \in \{1, 2, \ldots, 12\}$ accept x.

else: if $M_e(e)\downarrow$ then accept x.

else reject x.

let $f(e)$ be index of M. $e \in K \iff f(e) \notin L_{12}$

Claim 2: $K \leq_m L_{12}$

Proof given e, construct a TM M that does (given input x):

If $x \notin \{1, 2, \ldots, 12\}$ reject x

else if $M_e(e)\downarrow$ accept x

else reject x

let $f(e)$ be index of M. f is computable

$e \in K \implies M$ accepts $\{1, 2, \ldots, 12\} \implies f(e) \in L_{12}$

$e \notin K \implies M$ rejects all $\implies f(e) \notin L_{12}$

So L_{12} is not recursive.

$\overline{K} \leq L_{12}$, $\overline{K} \leq \overline{L_{12}}$ so L_{12}, $\overline{L_{12}}$ are not r.e.

Using $A \leq_m B \iff \overline{A} \leq_m \overline{B}$