Lec 1/26

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Reduction:

Dfn: Let $A, B \subseteq \omega$. We write $A \leq_m B$ if $\exists$ turing computable $f : \omega \to \omega$ such that $e \in A$ iff $f(e) \in B$ $\forall e \in \omega$.

We call f a (<u>many-to-one</u>) <u>reduction</u> from A to B

Some properties:

(i) $(A \leq_m B \ \& \ B \text{ is decidable}) \implies A$ is decidable.

(ii) $(A \leq_m B \ \& \ B \text{ is r.e.}) \implies A$ is r.e.

$(A \leq_m B \ \& \ A \text{ is decidable}) \not\implies B$ is decidable.

(iii) $(\underset{f}{A \leq_m B} \ \& \ \underset{g}{B \leq_m C}) \implies \underset{g \circ f}{A \leq_m C}$.

$\leq_m$ is <u>not</u> commutative

Let $K = \{ e : M_e(e)\downarrow \}$ $H_{\{\langle a,b\rangle : M_a(b)\downarrow\}}$, inputs

TMs	1	2	3	...	
1	0	1	1		
2	1	1	0		
3	1	0	0	1	1
⋮				0	
⋮					⋱

Diagonal has as much info as whole matrix

<u>Claim 1</u>: $K \leq_m H$

<u>Claim 2</u>: $H \leq_m K$

Pf: Let $f(e) = \langle e, e\rangle$. Then $e \in K$ iff $f(e) \in H$, and f is computable. □

Let $f(e) = \langle e, e \rangle$. Then $e \in K$ iff $f(e) \in H$, and f is computable. $\square$

Proof:

given e & x, construct a machine M:

M ignores its input, simulates M_e on x.

Recall:
$H = \{\langle e,x \rangle : M_e(x)\downarrow\}$

If $M_e(x)\downarrow$ then M accepts (M either accepts everything or loops on everything)

Define $f : \omega \to \omega$ as follows:

$f(\langle e,x \rangle) =$ the index of M above.

If $\langle e,x \rangle \in H$, then $f(\langle x,e \rangle)$ is the index of a TM which accepts everything

So $M_{f(\langle e,x \rangle)}(f(\langle e,x \rangle))\downarrow$ $f(\langle e,x \rangle) \in K$.

If $\langle e,x \rangle \notin H$ then $f(\langle e,x \rangle)$ is the index of a TM which loops on every input so $M_{f(\langle e,x \rangle)}(f(\langle e,x \rangle))\uparrow$ so $f(\langle e,x \rangle) \notin K$.

Thus $\langle e,x \rangle \in H \Leftrightarrow f(\langle e,x \rangle) \in K$ $\square$

Some non-recursive (undecidable) problems:

H, K

Some non-r.e. problems:

$\overline{H}$, $\overline{K}$