Lec 1/24

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Claim 4: If L and $\bar{L}$ are both r.e. then both are decidable.

Proof: Run recognizers for L & $\bar{L}$, if $x \in L$, $x \in L$. if $x \in \bar{L}$, $x \notin L$.

Notation: if $e \geq 1$ and $x \geq 0$ then we write $M_e(x)\downarrow$ if M_e accepts x.
otherwise we write $M_e(x)\uparrow$.

Define $H = \{\langle e, x\rangle : M_e(x)\downarrow\}$; H is called the halting problem

Is H r.e.? Is H recursive?

How to find "100th turing machine": check all strings in Lex-order.
if s is a valid encoding of a TM, increment counter.

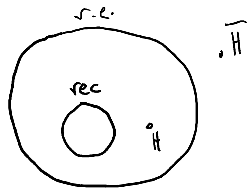
Theorem: H is not recursive.

Pf Assume it were. then $\exists\, n \in \mathbb{N}$ s.t. M_n decides H.
There is also an $i \in \mathbb{N}$ s.t. $M_i(x)$ simulates $M_n(\langle x, x\rangle)$, but accepts if M_n rejects and loops forever if M_n accepts.
is $\langle i, i\rangle \in H$? If so, i halts on input i so $M_n(\langle i, i\rangle)\downarrow$ but then i loops forever and so does not halt.
If not, i loops on input so M_n rejects $\langle i, i\rangle$, but then i accepts i and halts. In both cases $\times$

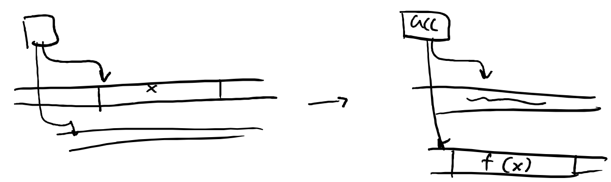
same proof

Pf2 Assume it was, then $\exists$ alg. A which decides H. Then $\exists$ alg. B which, given integer e, accepts if $M_e(e)\uparrow$ and rejects if $M_e(e)\downarrow$. By Church's thesis, $\exists$ TM that executes Alg. B. Let n be its index. If $M_n(n)\downarrow$ then B accepts n, i.e. $M_n(n)\uparrow$. If $M_n(n)\uparrow$ then B rejects n, i.e. $M_n(n)\downarrow$

Picture:



$f : \mathbb{N}^k \longrightarrow \mathbb{N}$ is computable if $\exists$



Non-computable: $\mathbb{1}_H$.

Let $g(x) = \begin{cases} 5 & \text{if Ghengis Khan is buried in Mongolia} \\ 6 & \text{o.w.} \end{cases}$

g is constant $\Rightarrow$ g is computable.