Lec 1/22

Monday, January 22, 2018 12:49

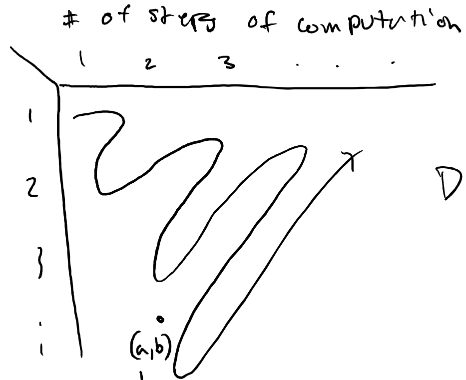
Let $\omega = \mathbb{N}$.

A problem is a subset of ω.

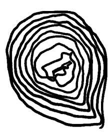
We'll write $W_e = L(M_e)$, Thus $W_1, W_2, \ldots$ are R.e. Languages.

Claim 1: L is r.e. $\iff$ $\exists$ an enumerator for L

Pf $\Rightarrow$



Dovetailing



Running forever is rejection.

(a,b) means run M(b) for a steps. (simulate)

if b is accepted eventually, it will be printed out.

And then not be printed again

$\Leftarrow$ run enumerator, if num is printed accept.

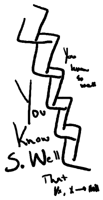
Claim 2: L is decidable $\iff$ L can be enumerated in Lexicographical order.

Notation If $L \subseteq \omega$ then $\bar{L} = \omega \setminus L$ is the complement of L.

True or False: $\forall L \subseteq \omega$, L is decidable iff $\bar{L}$ is.

?

True



True

$\forall L \subseteq \omega$, $\underbrace{L \text{ is R.E. iff } \bar{L} \text{ is R.E.}}$

$\Updownarrow$

Decidable = R.E.

(so it's false)

Dfn A function $f: \omega^k \longrightarrow \omega$ is (turing-)computable if $\exists$ a turing machine that, given input x, accepts x and leaves $f(x)$ on the output tape after computation.

Notation $(i,j) \mapsto \langle i, j \rangle$ is a computable 1-1 correspondence from $\omega \times \omega \longrightarrow \omega$

$$\langle i, j \rangle = \frac{(i+j-1)(i+j-2)}{2} + j$$

Likewise, $\langle i_1, \ldots, i_k \rangle$ is a computable 1-1 correspondence from $\omega^k \longrightarrow \omega$.