Lec 1/17

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Some cardinals: $0, 1, 2, \ldots$

$\aleph_0, \aleph_1, \ldots$

is there a cardinal α s.t. $\aleph_0 < \alpha < \aleph_1$?

Turing Machines: google: Kleene model for computation.

recursive function notation

Church-Turing Thesis:

Intuitive idea of computation is turing machine computation.

Discrete (In time steps)

Def: a turing machine is an 8-tuple

$M = \langle Q, \Sigma, \Gamma, q_0, q_{acc}, q_{rej}, B, \delta \rangle$

Q is a finite set of states

Σ " " of symbols, the input alphabet

Γ " ", the tape alphabet. $\Sigma \subset \Gamma$.

$q_0 \in Q$ is the start state

$q_{acc} \in Q$ is the accept state

$q_{rej} \in Q$ is the rejection state

$B \in \Gamma \setminus \Sigma$ is the blank symbol

$\delta : \Gamma \times (Q \setminus \{q_{acc}, q_{rej}\}) \to \{L, R\} \times \Gamma \times Q$ is the transition function.

A configuration of M is a pair $(q, x\bar{a}y)$
s.t. $q \in Q$, $a \in \Gamma$, and $x, y \in \Gamma^*$.

The first character of x cannot be a blank. Neither can the last of y.

A configuration specifies that M is in state q, and that the tape has $\leftarrow xay \rightarrow$ on it w/ R/W head scanning a.

... BBpoison $h\bar{a}$ve a sadcum BB ... not allowed.

Initial config: $(q_0, \bar{\alpha}_1 \alpha_2 \cdots \alpha_n)$ where $\alpha_i \in \Sigma$.

Def if C_1, C_2 are configurations of M then we write $C_1 \Rightarrow C_2$ in one step e.g. if $\delta(q_{42}, s) = (q_1, f, R)$

then $(q_{42}, \text{Shake}\bar{s}\text{peare}) \Rightarrow (q_1, \text{Shakef}\bar{p}\text{eare})$.

we write $C_1 \Rightarrow^* C_2$ if $\exists$ a finite set $\{C_a, C_b, \ldots, C_z\}$

$C_1 \overset{s.t.}{\Rightarrow} C_a \Rightarrow \cdots \Rightarrow C_z \Rightarrow C_2$.

M accepts $\alpha = \alpha_1 \alpha_2 \cdots \alpha_n \in \Sigma^*$ iff

$(q_0, \bar{\alpha}_1 \alpha_2 \alpha_3 \cdots \alpha_n) \Rightarrow^* (q_{accept}, x\bar{a}y)$ for some $x, y \in \Gamma^*$, $a \in \Gamma$.

otherwise M rejects α.

The Language accepted by M is $L_M = \{\alpha \in \Sigma^* : M \text{ accepts } \alpha\}$.