Lec 1/12

Friday, January 12, 2018 12:39

Def: $|A| \le |B|$ if $\exists$ 1-1 $f : A \to B$. (or onto $f : B \to A$).

$|A| = |B|$ if $|A| \le |B|$ and $|B| \le |A|$. Otherwise $|A| \ne |B|$.

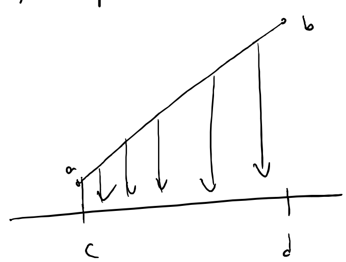
(or if $\exists$ 1-1 correspondence $f : A \to B$).

$|A| < |B|$ if $|B| \not\le |A|$. or if $|A| \le |B|$ but $|A| \ne |B|$.

Theorem: Let A, B be sets. Then exactly one of the following is true:

1) $|A| < |B|$ 2) $|B| < |A|$ 3) $|A| = |B|$

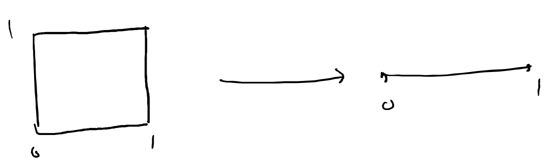
Claim: $|[a,b]| = \aleph_1$ $\forall a<b$. We know $|(0,1]| = \aleph_1$



tilt & project.

Now: $|\mathbb{R}| = |(a,b)|$.

Tangent function or something.



$f : [0,1)^2 \to [0,1)$

$f(a,b) = 0.a_1 b_1 a_2 b_2 \ldots$

for 1-1: ensure that a and b don't terminate in 7s,

So $f(a,b)$ won't either.

but $|\mathbb{R}^{\mathbb{R}}| > |2^{\mathbb{R}}| = \aleph_2$. what about $|\mathbb{R}^{\mathbb{N}}|$?

$= 2^{|\mathbb{N}|\cdot|\mathbb{N}|}$

$= 2^{|\mathbb{N}|} = \aleph_1$

Def Power set $POW(A)$ is

set of subsets of A

Thm $|A| < POW(A)$. $f: A \to POW(A)$ $f(a) = \{a\}$ is 1-1.

Proof if $g: A \to POW(A)$ is onto then $\forall X \subset A$, $\exists a \in A$ s.t. $g(a) = X$.

Let $S = \{a \in A : a \notin g(a)\}$. $\exists t \in A$ s.t. $g(t) = S$. is $t \in S$?

if $t \in S$ then $t \in g(t)$ so $t \notin S$, $\times$. if $t \notin S$ then $t \notin g(t)$

So $t \in S$ $\times$. thus in both cases we have $\times$ so

g must not be onto.