Lec 1/10

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$\mathbb{Q}$ is countable

Lemma: if $\exists f: S \to \mathbb{Z}^+$ 1-1 then S countable

Pf: If S is finite then S is countable. Assume S infinite.

Let $f(S) = \{i_1 < i_2 < i_3 < \cdots\} \subseteq \mathbb{Z}^+$. then let $g(i_n) = n$. g is 1-1 and onto.

So $g \circ f$ is a 1-1 correspondence between S and $\mathbb{Z}^+$. ∎

Defs A Partial ordering on a set S is a binary relation $R \subseteq S \times S$ written $x \leq y$ iff $(x, y) \in R$, where the following properties hold:

(i) $x \leq x$

(ii) if $x \leq y$, $y \leq z$, then $x \leq z$

(iii) if $x \leq y$, $y \leq x$, then $x = y$.

A total ordering is a partial ordering where $\forall x, y \in S$ either $x \leq y$ or $y \leq x$.

Theorem: $[0,1)$ is uncountable.

Pf: Diagonalization Argument

The cardinality of $[0,1)$ is $\aleph_1$ or $\mathbf{c}$. or $|\mathbb{R}|$

$\mathbb{Z}^+$ is $\aleph_0$

all the cardinals:

$$0, 1, 2, \ldots$$

$$\aleph_0, \aleph_1, \aleph_2, \ldots$$